

TIME-SERIES SYNTHESIS OF DELAY LINE
COMPENSATORS

By

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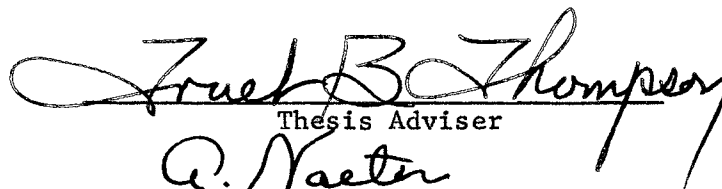
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CHAPTER I

GENERAL INTRODUCTION

PART I

DELAY TYPE COMPENSATION

The method of using a compensator in conjunction with a controlled system has long been practised by control system designers in order to obtain a desirable steady state or dynamic performance. However, in the past, only passive networks were used. So far as the purpose of compensation is concerned, most of the developments are to stabilize an unstable system or to improve the steady and/or the transient state performance of a system.

The use of a delay-type device (or the short-time-memory device) as a compensator to form a follower-type control system, in which it is desired to cause the output to follow or match the input at all times as closely as possible, has been proposed by several authors in recent years. (1-6).

According to the physical conception, an ideal follower-type control system can not be made by using a passive network. This is based upon the fundamental conception of dissipation of energy stored in the system. But it is possible to make an active network such as a delay-line device meet such requirements. This is shown to be true by research and experiments which will be described later.

(I) Historical Background

In 1940, H. E. Kallman (1) devised the first delay-line filter, which he called the "Transversal Filter" to distinguish it from the conventional filter. Kallman assumed in his paper, that the delay-line was perfect with smooth energy flow, no internal dissipation, and no reflections.

As shown in Fig. 1, Kallman's delay line consisted of a number of small section of lumped-constant filters. It is terminated by a resistance equal to the characteristic impedance of the line to eliminate reflection. Along the delay line numerous tapping points are provided to secure a signal of specified time delay. The input signal passes from the input terminal and is propagated along the line. The energy is largely dissi-

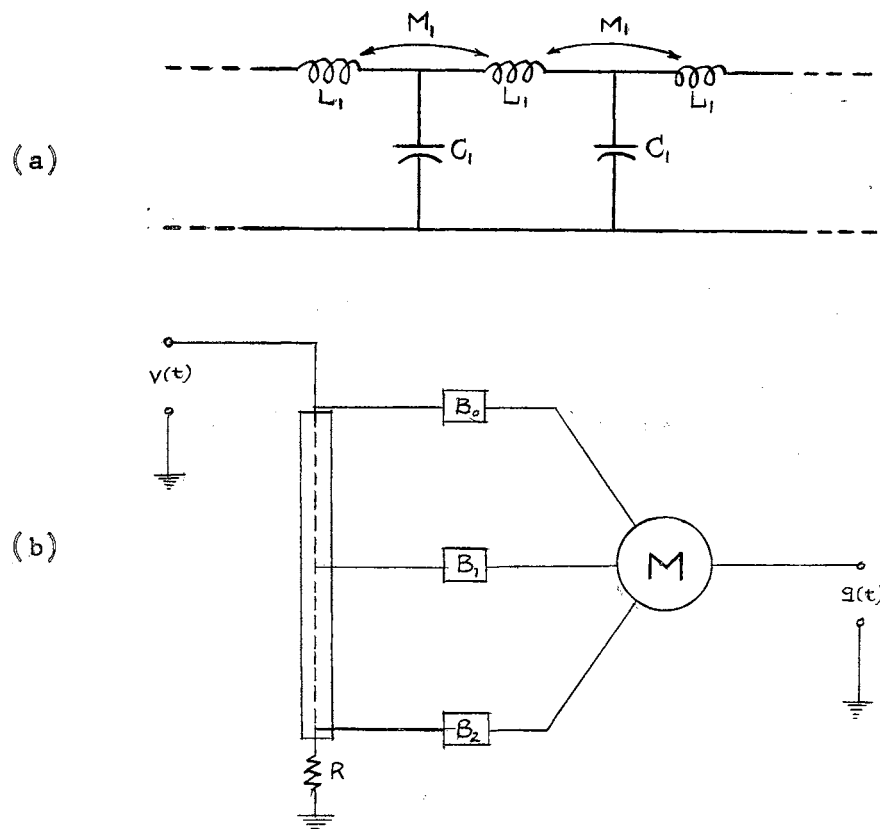


Fig. 1 The Transversal Filter

pated in the terminating resistance. Small portions of energy are tapped from the line at the various tapping points. After amplification and possible change of polarities, the amplified signals are added together at a summing point. The desired response can be achieved by designating different values of amplification for the amplifiers.

However, the purpose of the above particular work was to produce a filter which had the required amplitude and phase characteristics and which, within itself, produced a linear phase-lag response. Apparently no attempt was made to establish the necessary design procedure or to utilize that network as a compensator for operation in combination with a control system.

The work of A. M. Hopkin (2) in 1951, describes the use of an amplified command signal plus two delayed and amplified signals to control a non-linear second order system under the conditions which would be imposed by a step function command signal.

The design of the delay-type compensator, named the "Signal Component Control Compensator" was described by J. F. Calvert and D. J. Gimpel (3) in 1952. In that paper, a polynomial command time function is used. The design was based on the Laplace transformation. They made the application of step command to first and second order control system. The result is that the transient response will quickly follow the command and all the natural modes of oscillation are removed.

In 1954, J. F. Calvert and D. J. Ford (4) published a paper based on the latter's Ph.D. dissertation. In this paper they described three new types of delay line compensators with all analysis in the frequency domain:

- (1) Taylor's Series Compensator (TSC)----- For polynomial command signal.
- (2) Fourier Series Compensator (FSC)----- For sinusoidal command signal.
- (3) Fourier Integral Compensator (FIC)----- For sinusoidal command signal.

In 1955, J. F. Calvert and T. W. Sze (5) published a paper discussing a general application of short time memory devices. A satisfactory result was given when this device was used on the following system:

- (1) Open loop circuit.
- (2) Closed loop circuit.
- (3) Feedback circuit.
- (4) Feedforward circuit.
- (5) The circuit of the combination of (3) and (4).

A more important document related to this report is the Ph.D. dissertation of Dr. Truet B. Thompson (6) in which he derived a new method for the design of a delay-type compensator. This method is based in the time domain, using time series. By applying this method he work the same problem as the three previous authors did. The result showed that the time series method is shorter and more direct than those which used frequency analysis, and the results are equally good.

In this report, the method of analysis is based on the Thompson scheme. The author applies this method to synthesize the design of delay line compensators, for various orders of systems, with various types of command time functions. The procedure will be stated in the end of this chapter and the details will be seen in Chapter II.

(II) Function of Delay Line Compensator

The function of a delay line compensator may be demonstrated as shown in the following diagram:

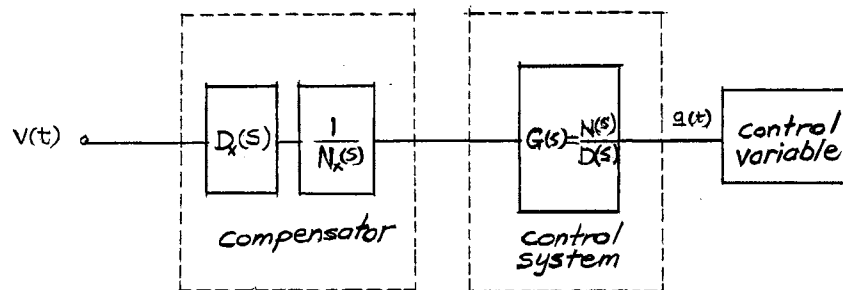


Fig. 2 Block Diagram of Compensated System

As shown in Fig. 1, the delay line itself is composed of a number of small sectional lumped-constant filters. A resistance which is equal to the characteristic impedance of the line is used at the end of the line to eliminate reflection. Along the delay line a number of taps are provided to secure signals of specific delay.

The input signal passes from the input terminal and is propagated along the line, and the energy is largely dissipated in the terminating resistance.

When the first sampled signal enters through the first terminal and is amplified by the amplifier B_0 , then this amplified signal is transmitted to the mixing device. Through the compensator component it produces a compensating action to the control system.

Before the second signal passes in, the whole circuit will be under the domain of the first. By the same manner, each succeeding signal passes through the system. When steady state is reached, the output will follow the input in direct correspondence.

PART II

THE TIME SERIES

The method of synthesizing network problems can be classified into two families. One is frequency analysis and the other is time domain. It is well known that the Fourier Integral, Laplace-Transformation and many other theorems found in the textbooks most were based on frequency analysis. (7-9). The trend to use time domain has grown stronger in recent years. The reason is that the time domain method is considered more suitable to some particular problems. The time series operation in conjunction with the design of delay line compensators belongs entirely to the method of time domain.

The time series was first employed by A. Tustin (10) in 1947. As he stated in his paper, he used a polynomial to describe a time function. Such a polynomial is called a time series. Using the technique of polynomial manipulation, he synthesized the problem of linear systems with time series.

W. H. Huggins (11), in his paper, used Taylor's series expansion to determine the system transfer function for impulse response. His method laid the foundation of later time domain development.

In 1950, a Ph.D. dissertation presented by A. Madwed (12) made the application of time series to the solution of differential equations. Time series proved to be a useful tool in network synthesis.

Two more papers announced in 1954 by Freddy Ba Hli (13) and J. G. Truxall (14) introduced the use of time series for synthesis.

R. Boxer and S. Thaler (15) published a paper in 1956 using z -transform and synthetic division finding the time response directly when the system transfer function was given. The response resulting was

in the form of a time series. It shows that time series can be applied to analysis problems.

In 1956, Thompson's paper (16) introduced a method for the design of delay line compensators using time series.

In this paper the author applies this method in the synthesis of the delay line compensators. The definition and manipulation of time series is shown in Appendix A; more detailed derivation of time series calculus and the relations between time series and other transformation methods can be found in reference 6 of the Bibliography.

There are two important points concerning the design method. One is that all the responses must be expressed in time series. The methods of getting time series expressions for a system response when system transfer function is given is shown in Appendix C. The second point is that the design is based on the polynomial operation of time series. The fundamental relation will be shown later.

The relations among the time series and the compensator design can be simply related as follows:

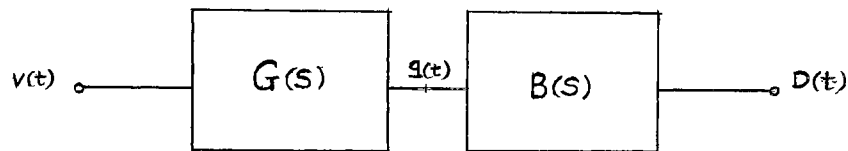


Fig. 3 Schematic Diagram of Compensation

Referring to the above figure, $v(t)$ is the system command function; and $q(t)$ is the response of the system. According to the Theorem of Laplace-Transformation, the function relating the input and output is called the system transfer function. Let the transfer function of the

system be $G(s)$, and express the relation in Laplace form that is:

$$G(s) = \frac{q(s)}{v(s)} \quad (1-1)$$

where s is the Laplace variable.

If the compensator is connected in cascade to the control system as shown in Fig. 3, the transfer function will be:

$$B(s) = \frac{D(s)}{q(s)} \quad (1-2)$$

where $D(s)$ is the prescribed desired response which actually is the input command function itself.

For this particular method, the compensator designed is to use the relation much like the one shown above, that is:

$$B(x) = \frac{D(x)}{Q(x)} \quad (1-3)$$

the special feature of the above equation is that the $D(x)$ and $Q(x)$ are all the time series expressions of the desired and uncompensated responses respectively.

The result of this design is also in the form of a time series. This time series $B(x)$ is defined as the successive gains of the amplifiers of the delay line compensator.

PART III

GENERAL SCOPE OF STUDY

(I) The Design Criterion

A general delay line compensation schematic diagram is shown in Fig. 4.

The control system under consideration has a transfer function which is:

$$G(s) = \frac{N(s)}{D(s)} \quad (1-4)$$

where s is the Laplace variable, $N(s)$ and $D(s)$ are the numerator and denominator of $G(s)$ respectively; both are polynomial of s . Usually the order of $N(s)$ is lower than $D(s)$.

The compensator, for convenience, is considered to consist of two parts cascaded as:

$$B(s) = \frac{D_x(s)}{N_x(s)} \quad (1-5)$$

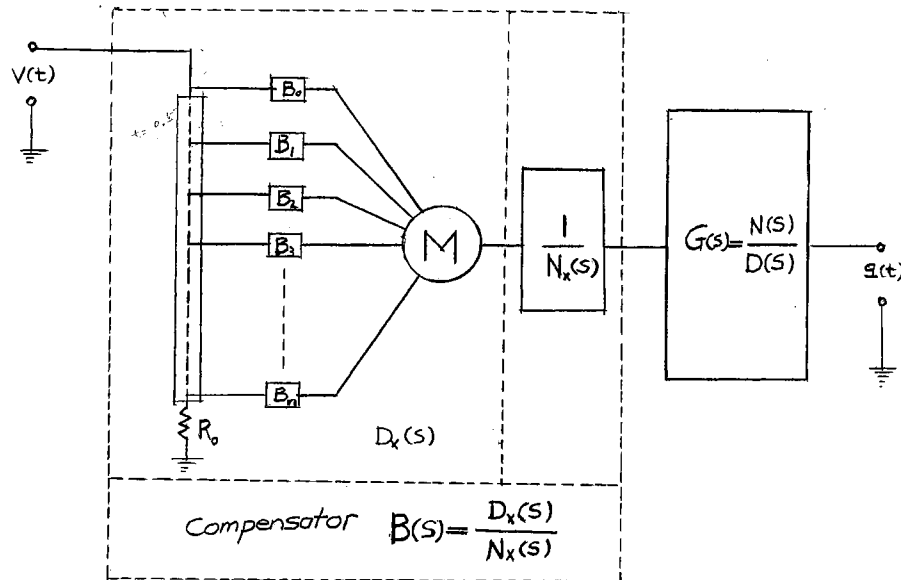


Fig. 4 Delay Line Compensation

Thus the system overall transfer function becomes:

$$F(s) = \frac{q(s)}{v(s)} = \frac{N(s)}{D(s)} \cdot \frac{D_x(s)}{N_x(s)} \quad (1-6)$$

It is possible by conventional network design to produce a transfer function of $\frac{1}{N_x(s)}$ where $N_x(s)$ is equal to $N(s)$. Such network generally takes the form of a cascaded passive network. This means to produce a transfer function having poles matching the zeros of $N(s)$, which gives:

$$F(s) = \frac{q(s)}{v(s)} = \frac{D_x(s)}{D(s)} \quad (1-7)$$

Thus the design of the compensator will be concerned entirely with the design of $D(s)$. From Eq. (1-7) we know that, if the compensated system is to be an ideal system, $F(s)$ should be equal to unity. So that the instantaneous difference between the command function and the output signal is zero at all times. This idealization is never possible by any practical method.

The next best objective would be to remove all modes of oscillation and periodic errors in the transient response.

It is well known that the denominator of the transfer function $F(s)$ is the characteristic equation of the system, that is:

$$\begin{aligned} D(s) &= a_0 + a_1s + a_2s^2 - - - - - + a_ns^n \\ &= a_n (s-u_1)(s-u_2) - - - - - (s-u_n) \end{aligned} \quad (1-8)$$

where u 's are the roots of the characteristic equation and poles in the complex plane (8) of the control system. The order of the system is given by n .

Corresponding to a step command the generalized form of the transfer function of a delay line compensator (5) is:

$$B(s) = B_0 + B_1 e^{-sT_1} + \dots + B_n e^{-sT_n} \quad (1-9)$$

where n is the order of the system, and the exponents T 's are the time delays which are positive real quantities, and the coefficient B 's are the amplifier gains which are either positive or negative real numbers.

The system transfer function is now:

$$\frac{B_0 + B_1 e^{-sT_1} + B_2 e^{-sT_2} + \dots + B_n e^{-sT_n}}{a_0 + a_1 s + a_2 s^2 + \dots + a_n s^n} \quad (1-10)$$

the numerator $N(s)$ of $G(s)$ is assumed to be removed by the use of $1/N(s)$.

It is desired now that the final value of the response to a unit step function to be unity, thus by the final value theorem we have:

$$\begin{aligned} 1 &= \lim_{s \rightarrow 0} \frac{s (B_0 + B_1 e^{-sT_1} + \dots + B_n e^{-sT_n})}{s (a_0 + a_1 s + \dots + a_n s^n)} \\ &= (B_0 + B_1 + B_2 + \dots + B_n) / a_0 \end{aligned} \quad (1-11)$$

$$\text{hence } (B_0 + B_1 + B_2 + \dots + B_n) = a_0 \quad (1-12)$$

This equation is defined as a set of amplifier gains for the delay line compensator.

For a particular case, when a_0 is equal to unity, Eq. (1-12) reduces to:

$$\sum_{k=0}^n B_k = 1 \quad (1-13)$$

this criterion is deduced for step input. And it can be held for all kinds of input functions, upon proper treatment of the design of $D(s)$.

(II) The System Transfer Function

The generalization of the synthesis mainly depends upon the system

used. Through this synthesis, three different kinds of systems are to be used, such as first order, second order and third order systems. (It will shown later than this synthesis can be extended to any higher order system.)

In choosing the control system, the following two conditions must be considered:

- (1) This system must be as general as possible.
- (2) This system is considered to be stable. (That is, no poles or zeros containing positive real parts in the system complex plane).

Fortunately, a second order system, which has been used by four authors (3-6) is available and can be used as a check to the previous studies.

By the similar ways we select our suitable system in first and third order as listed below:

TABLE I

First Order	Second Order	Third Order
$\frac{1}{(s + 1)}$	$\frac{1}{(s^2 + 0.8s + 1)}$	$\frac{1}{(s^3 + 1.8s^2 + 1.8s + 1)}$

(III) The Command Function

This synthesis contains a series of calculations using five different kinds of command time functions. These time functions can easily be found in any common servomechanism book. The time functions and their Laplace Transforms are shown in Table II.

TABLE II

Time Function	Heaviside Expression	Laplace-Transform
Unit Impulse	$\delta(t)$	1
Unit Step	$U(t)$	$1/s$
Ramp Function	$t \cdot U(t)$	$1/s^2$
$t^2/2!$	$t^2/2! \cdot U(t)$	$1/s^3$
$t^3/3!$	$t^3/3! \cdot U(t)$	$1/s^4$

(IV) The Specification of the Delay Line

For the sake of simplicity throughout the synthesis, the delay interval between taps along the delay line at $t = 0.5$ second. That is, when four amplifiers are used, the length of the delay line should be 1.5 seconds.

(V) The Method of Study

The method involved in this synthesis may be stated by the following:

(1) This design method started from a given uncompensated response. For a certain system and a given command signal, the response can be obtained by the following methods:

- (a) Use Laplace Transformation method to get the time function; calculate in tabulated form by the substitution of t .
- (b) Use the system response transfer function and apply the method of Boxer and Thaler (15) expanding it into time series.

(c) Copy the response or read it directly from the oscillograph or other graphical recorder.

(2) For the follower-type compensating system, after all the modes of oscillation and periodic errors are removed, the output signal will follow the input instantaneously. Thus the desired response is approximately the same as the input command signal, except for a very small transient period (i.e. before the follower action becomes steady). In this process the so-called transient response is not defined. In order to get the proper results, it is necessary to preassign some suitable values for the desired response.

(3) To write the time series expression of the desired and the uncompensated responses, use the same time interval (say 0.5 second). Choose sufficient terms of the time series.

(4) Apply polynomial division of the two time series as follows:

$$\begin{array}{r} D_1x + D_2x^2 + D_3x^3 + \dots + D_mx^m \\ \hline q_1x + q_2x^2 + q_3x^3 + \dots + q_mx^m \\ \hline = B_0 + B_1x + B_2x^2 + \dots + B_kx^k \end{array} \quad (1-14)$$

the process based on approximation, both polynomials are arranged in the manner of ascending power of x . The division carries on until the remainder of the division approaches zero.

(5) The coefficients B are the amplification factors of these amplifiers of the compensator. The number of stages of the amplifiers determines the length of the delay line. The relation is:

$$L = (N + 1)v \quad (1-15)$$

where L is the length of the delay line in seconds, N is the number of amplifiers, and v is the delay length between taps in seconds.

(6) Check the coefficients of the polynomial $B(x)$ with the basic design criterion:

$$\sum_{k=0}^n B_k = 1 \quad (1-13)$$

if the answer does not confirm with the prescribed condition some necessary modification or a new design is needed.

(7) In completing the design, a verification procedure is needed. Using time series multiplication to multiply out the $B(x)$ with $Q(x)$ yields the desired response. The accuracy of the result will be seen from a comparison of the designed response to the prescribed response. An ideal compensator can be obtained if proper treatment is applied.

CHAPTER II

TIME SERIES METHOD

PART I

BASIC TIME SERIES METHOD

As stated before, the principle of compensation is based upon the elimination of the poles of the control system transfer function. So far as the time series method is concerned, there is no need to find the locations of these poles. The whole process acts as a short cut for the convolution of two time functions. However, the convolution process is not so apparent when using time series. This will be seen in the following illustration.

The method of design is described in Chapter III. An outline of this method will be given below:

- (a) Find the time series expression of the uncompensated time response.
- (b) Choose the desired response and express it in time series form.
- (c) Determine the co-efficients of the amplifiers of the transversal filter circuit by time series division of the desired and uncompensated response.
- (d) Verification of design.

Taking a second order system with a step command function as an

example, the design procedure can be demonstrated as follows:

(1) Uncompensated Response:

From TABLE I and TABLE II, we can get the overall system transfer function of this system is:

$$q(s) = \frac{1}{s(s^2 + 0.8s + 1)} \quad (2-1)$$

in order to obtain a more accurate result the time series evaluated here is based on Laplace Transformation method; the effective values calculated up to eight places. A set of calculated data is listed in TABLE III. The response is plotted as shown in Fig. 13, Curve A.

(2) The Desired Response:

The desired response of this system is shown in Curve B of Fig. 5. When the time is below one second, the curve is a dotted line, which means during the transit period the value is undefined. In this case, it is assumed to be a straight line. The ordinate at time equals 0.5 second is assumed to be 0.5 unit.

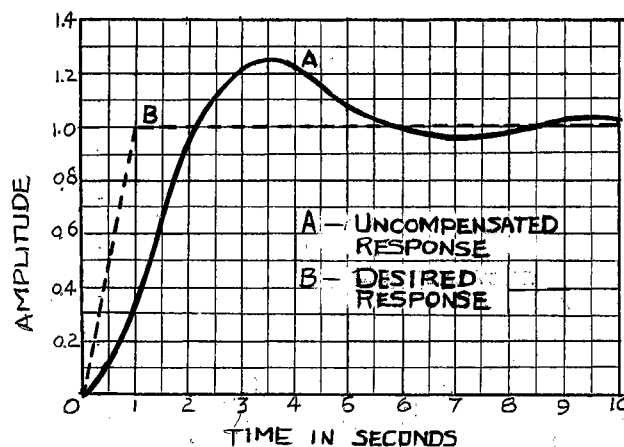


Fig. 5 Response Curves of Second Order System with Unit Step Input

(3) Time Series Division:

The time series of the uncompensated response $Q(x)$ is:

TABLE III

Responses of Second Order System With Step Input

Time (sec.)	Exact Solution	Boxer & Thaler Solution	Compensated* Response
0.0	0.00000	0.00000	0.00000
0.1	0.00487		0.02410
0.2	0.01889		0.09360
0.3	0.04129		0.20470
0.4	0.07117		0.35280
0.5	0.10766	0.10239	0.53720
0.6	0.14992		0.70770
0.7	0.19709		0.83940
0.8	0.24834		0.93030
0.9	0.30287		0.98300
1.0	0.35991	0.35504	1.00000
1.1	0.41878		
1.2	0.47872		
1.3	0.53896		
1.4	0.59900		
1.5	0.65827	0.65698	0.99999
1.6	0.71627		
1.7	0.77253		
1.8	0.82666		
1.9	0.87828		
2.0	0.92711	0.93023	0.99930
2.1	0.97288		
2.2	1.01540		
2.3	1.05449		
2.4	1.09006		
2.5	1.12207	1.12823	0.99930
3.0	1.22812	1.23510	0.99930
3.5	1.25319	1.25881	0.99960
4.0	1.21889	1.22175	0.99980
4.5	1.15172	1.14142	1.00000
5.0	1.07609	1.07312	1.00000

* Compensated Response use $B_0 = 4.9573$, $B_1 = -7.2838$, $B_2 = 3.3265$

$$Q(s) = q_0 + q_1x + q_2x^2 + - - - - - + q_nx^n \quad (2-2)$$

and for the desired response is:

$$D(x) = 0.5x + x^2 + x^3 + - - - - - + x^n \quad (2-3)$$

where n is positive integer. The value of n required is dependant upon the necessity to get a definite series B(x).

The division is related by the following:

$$\begin{aligned} B(x) &= \frac{D(x)}{Q(x)} \\ &= \frac{0.5x + x^2 + x^3 + - - - - - + x^n}{q_1x + q_2x^2 + q_3x^3 + - - - - - + q_nx^n} \\ &= B_0 = B_1x + B_2x \end{aligned} \quad (2-4)$$

where the co-efficients of Q(x) and B(x) are:

$q_1 = 0.10239$	$q_6 = 1.23510$
$q_2 = 0.35504$	$q_7 = 1.25881$
$q_3 = 0.65698$	$q_8 = 1.22175$
$q_4 = 0.93023$	$q_9 = 1.14142$
$q_5 = 1.12823$	$q_{10} = 1.07312$

$$B_0 = (+) 4.883$$

$$B_1 = (-) 7.166$$

$$B_2 = (+) 3.283$$

This polynomial division ended with all B's higher than B_2 equal to zero. Therefore, only three amplifier are needed along the delay line.

(4) Verification of Design:

From the above result, the sum of the co-efficients B is approximately equal to unity. This result is confirmed with our design

criterion.

With a further consideration, a verification of the above design is made by multiplication of the two cascaded system. The final form of the compensated system shows that Curve C at the time 0.5 second is not passing through the point of argument. The actual value at that time is 0.5257. Then recalculate $B(x)$ using 0.5257 instead of 0.5 which finally yields:

$$B(x) = 4.9573 - 7.2838x + 3.3265x^2 \quad (2-5)$$

A better approximation is obtained by this modification and the compensated response with the use of these new constants is plotted in Fig. 6.

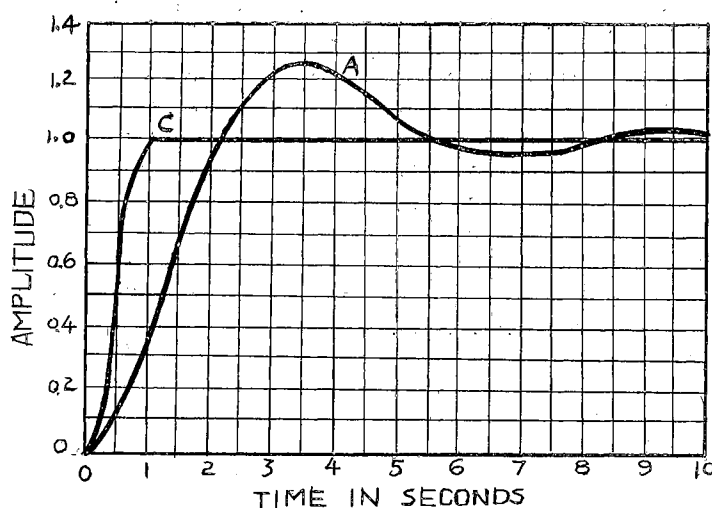


Fig. 6 Compensated Characteristic
of Second Order System

A - Uncompensated Response
C - Compensated Response

PART II

A MODIFICATION OF THE METHOD

The last example shown how the time-series division relates the desired and the uncompensated responses. So far as the more general problem is concerned, the method may be difficult. Some modifications help the method work adequately. One of the modifications is called, "The Amplitude Constraint Method". (16). This method is probably the simplest one to be introduced.

From the polynomial division of Eq. (2-4) an inverse operation yields an important relation between those coefficients which can be readily written as the following:

$$D(x) = B(x) \cdot Q(x) \quad (2-6)$$

suppose these time series take the forms:

$$D(x) = d_0 + d_1x + d_2x^2 + \text{-----} + d_nx^n \quad (2-7)$$

$$Q(x) = q_0 + q_1x + q_2x^2 + \text{-----} + q_nx^n \quad (2-8)$$

and the time series $B(x)$ has definite terms

$$B(x) = B_0 + B_1x + B_2x^2 + \text{-----} + B_kx^k \quad (2-9)$$

where $k < n$. The n and k may be any positive integers. Multiply them out in terms of their coefficients. Then Eq. (2-6) becomes:

$$D(x) = (B_0 + B_1x + B_2x^2 + \text{-----} + B_kx^k) \cdot (q_0 + q_1x + q_2x^2 + \text{-----} + q_nx^n)$$

$$\begin{aligned}
&= B_0 q_0 + (B_0 q_1 + B_1 q_0)x \\
&\quad + (B_0 q_2 + B_1 q_1 + B_2 q_0)x^2 \\
&\quad + (B_0 q_3 + B_1 q_2 + B_2 q_1 + B_3 q_0)x^3 \\
&\quad + \text{-----} \\
&\quad + (B_0 q_k + B_1 q_{k-1} + \text{-----} + B_k q_0)x^k \\
&\quad + \text{-----} \\
&\quad + (B_0 q_n + B_1 q_{n-1} + \text{-----} + B_k q_{n-k})x^n
\end{aligned} \tag{2-10}$$

Comparing the coefficients of Eq. (2-7) with those of Eq. (2-10) gives the set of useful relationships shown below:

$$\begin{aligned}
d_0 &= B_0 q_0 \\
d_1 &= B_0 q_1 + B_1 q_0 \\
d_2 &= B_0 q_2 + B_1 q_1 + B_2 q_0 \\
d_3 &= B_0 q_3 + B_1 q_2 + B_2 q_1 + B_3 q_0 \\
&\text{-----} \\
d_k &= B_0 q_k + B_1 q_{k-1} + \text{-----} + B_k q_0 \\
&\text{-----} \\
d_n &= B_0 q_n + B_1 q_{n-1} + \text{-----} + B_k q_{n-k}
\end{aligned} \tag{2-11}$$

It is evident that the d 's are the coefficients of the desired response. When the desired response is chosen, these values are known since all q 's are known quantities. From Part IV of Chapter III, we know the value of the desired response in the very short transient period is undefined; but it should be defined because those early terms, especially the leading term, are very important for the determination of the correct amplifier gains. Fortunately, the useful relations listed in Eq. (2-11) can be used to determine those unknown values. The method is to write a set of simultaneous equations which relate the known quantities to the unknown.

Solve these equations to get enough information to define the desired response. Then the design method of time series division may be applied.

This method shows its greatest advantage in the design of higher order system compensators. Illustrated below is the constraint method of design for a second order system with ramp input. Such a system can be compensated by using four amplifiers with a delay line length of one and a half seconds. The time response of this system is tabulated in TABLE IV, a time series can readily be written as the following:

$$\begin{aligned} Q(x) = & 0.01873x + 0.13177x^2 + 0.38616x^3 + 0.78485x^4 \\ & + 1.30072x^5 + 1.89194x^6 + 2.51525x^7 + 3.13519x^8 \\ & + 3.72867x^9 + 4.28554x^{10} \end{aligned} \quad (2-12)$$

And the desired response is:

$$\begin{aligned} D(x) = & d_1x + d_2x^2 + 1.5x^3 + 2.0x^4 + 2.5x^5 + 3.0x^6 \\ & + 4.0x^7 + 4.5x^8 + 5.0x^9 + 5.5x^{10} \end{aligned} \quad (2-13)$$

d_1 and d_2 are to be determined by the following set of equations:

$$\begin{aligned} 0.38616B_0 + 0.13177B_1 + 0.01873B_2 + 0.00000B_3 &= 1.5 \\ 0.78485B_0 + 0.38616B_1 + 0.13177B_2 + 0.01873B_3 &= 2.0 \\ 1.30072B_0 + 0.78485B_1 + 0.38616B_2 + 0.13177B_3 &= 2.5 \\ B_0 + B_1 + B_2 + B_3 &= 1.0 \end{aligned} \quad (2-14)$$

The upper three equations are based upon Eq. (2-11) and the lower is based upon the design criterion. In this case we need just to solve B_0 and B_1 only.

$$\begin{aligned} B_0 &= 9.742112 \\ B_1 &= (-) 19.09317 \end{aligned}$$

Substitute B_0 and B_1 into Eq. (2-11) and then

$$\begin{aligned} d_1 &= 0.182414 \\ d_2 &= 0.925991 \end{aligned}$$

TABLE IV
SECOND ORDER SYSTEM WITH RAMP INPUT

Time (Sec.)	Command Signal	Uncompensated Responses	Compensated Response
0.0	.000000	0.000000	0.000000
.1	.100000	0.000209	0.002040
.2	.200000	0.001327	0.027927
.3	.300000	0.004273	0.041619
.4	.400000	0.009839	0.095824
.5	.500000	0.018729	0.182450
.6	.600000	0.031564	0.303422
.7	.700000	0.048876	0.450709
.8	.800000	0.071117	0.611102
.9	.900000	0.098653	0.773080
1.0	1.000000	0.131773	0.926152
1.1	1.100000	0.170690	1.062929
1.2	1.200000	0.215545	1.184549
1.3	1.300000	0.266411	1.295441
1.4	1.400000	0.323299	1.399402
1.5	1.500000	0.386157	1.500000
1.6	1.600000	0.454885	1.599903
1.7	1.700000	0.529331	1.699936
1.8	1.800000	0.609301	1.799962
1.9	1.900000	0.694563	1.899984
2.0	2.000000	0.784851	1.999999
2.5	2.500000	1.300723	2.500000
3.0	3.000000	1.891939	2.999911
3.5	3.500000	2.515251	3.499780
4.0	4.000000	3.135194	3.999648
4.5	4.500000	3.728671	4.499540
5.0	5.000000	4.285545	4.999471
5.5	5.500000	4.806406	5.499445
6.0	6.000000	5.298915	5.999448
6.5	6.500000	5.773933	6.499479
7.0	7.000000	6.242266	6.999499
7.5	7.500000	6.712510	7.499549
8.0	8.000000	7.190039	7.999569
8.5	8.500000	7.676987	8.499594
9.0	9.000000	8.172882	8.999596
9.5	9.500000	8.675602	9.499596
10.0	10.000000	9.179206	9.999569

Substitute d_1 and d_2 into Eq. (2-13) and a modified desired response is obtained. Use this time series divided by $Q(x)$ yields:

$$B(x) = 9.742112 - 19.093017x + 13.559651x^2 - 3.208748x^3$$

This equation describes the positions and the quantities of the amplifiers.

TABLE V

Amplifiers	Positions	Polarity	Amplifier Constant
B_0	0.00 Sec.	(+)	9.742112
B_1	0.5 Sec.	(-)	19.093017
B_2	1.0 Sec.	(+)	13.559651
B_3	1.5 Sec.	(-)	3.208748

Verification of this design gave the result as shown in Fig. 7, Curve C.

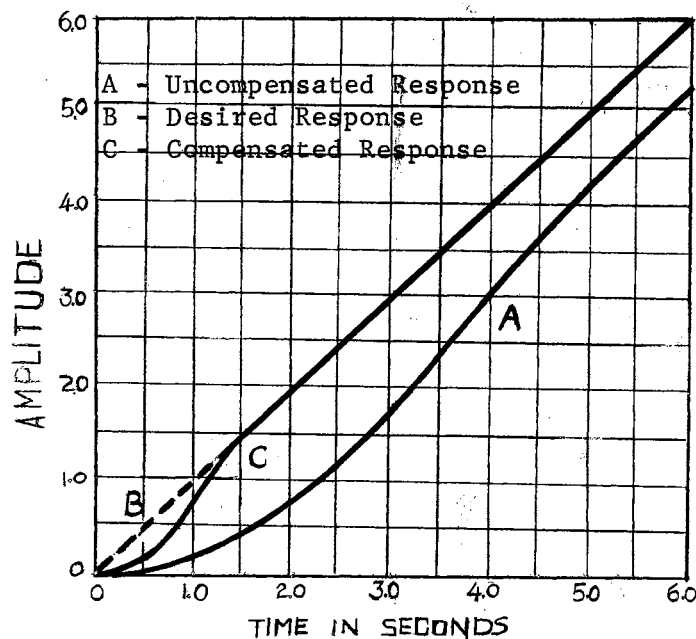


Fig. 7 Response Curves of a Second Order System With Ramp Input

PART III

SUMMARY OF THE SYNTHESIS

This work includes three kinds of control system and five kinds of command time functions. Fifteen cases were evaluated. An outline of this synthesis is summarized as follows:

- (1) The types of control systems are listed in TABLE I.
- (2) The types of command time functions are listed in TABLE II.
- (3) The solution of uncompensated responses as time functions are listed in TABLE X - XII in Appendix D.
- (4) The calculated values of uncompensated, desired and compensated responses are listed in TABLE XIII - XXV of Appendix D.
- (5) The required amplification factors are listed in TABLE VI-VIII.
- (6) The responses curves of each system were plotted as shown in Fig. 8 to Fig. 20.

For the unit impulse input the design method is simply the basic time series method. For any other higher order input the design is based on the modified method.

TABLE VI
LIST OF AMPLIFICATION COEFFICIENTS
FIRST ORDER SYSTEM

	${}_1B_0^*$	${}_1B_1$	${}_1B_2$	${}_1B_3$	${}_1B_4$
B_0	+ 6.107014	3.033900	+ 8.183097	+ 9.931281	+ 10.853358
B_1	- 5.000000	- 2.033900	- 9.366194	- 14.293820	- 17.448136
B_2			+ 2.183097	+ 6.793795	+ 10.475844
B_3				- 1.431256	- 2.888507
B_4					+ 0.007441

* nB_k the amplifier factor of a specific system, where n denotes the order of the control system, and k is the minus power of Laplace variable (S) of the command function. For example, ${}_1B_0$ is the amplification factor of the first order system with unit impulse input.

TABLE VII
LIST OF AMPLIFICATION COEFFICIENTS
OF SECOND ORDER SYSTEM

	$2B_0$	$2B_1$	$2B_2$	$2B_3$	$2B_4$
B_0	+ 5.060916	+ 4.957300	+ 9.742112	+ 14.030886	+ 16.055847
B_1	- 7.432011	- 7.283800	-19.093017	- 33.962314	- 41.344773
B_2	+ 3.392482	+ 3.326500	+13.559651	+ 33.299858	+ 42.447238
B_3			- 3.208748	- 15.235930	- 17.981473
B_4				+ 2.867536	+ 0.001677 *
B_5					+ 1.821484

* See Discussion (4), Chapter III.

TABLE VIII

LIST OF AMPLIFICATION COEFFICIENTS
OF THIRD ORDER SYSTEM

	$3B_0$	$3B_1$	$3B_2$	$3B_3$	$3B_4$
B_0	+10.961667	+ 11.673140	+ 30.608858	+ 50.725856	+ 52.145422
B_1	-22.695814	- 23.486423	- 81.583580	-163.594350	-153.572590
B_2	+16.967455	+ 16.939101	+ 85.311557	+220.457390	+151.155590
B_3	- 4.233080	- 4.125818	- 40.711447	-153.671140	+ 0.000217 *
B_4			+ 7.374609	+ 55.321330	-113.341660
B_5				8.239068	+ 87.346066
B_6					- 22.733028

* See Discussion (4), Chapter III.

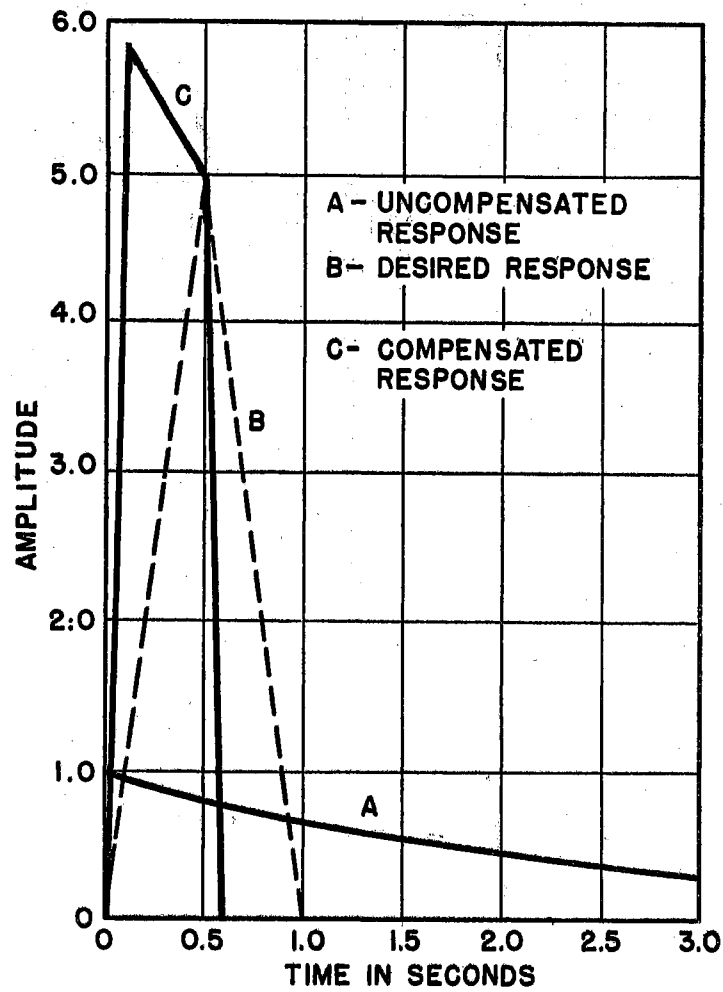


Fig. 8 Response Curves of First Order System With Unit Impulse Input

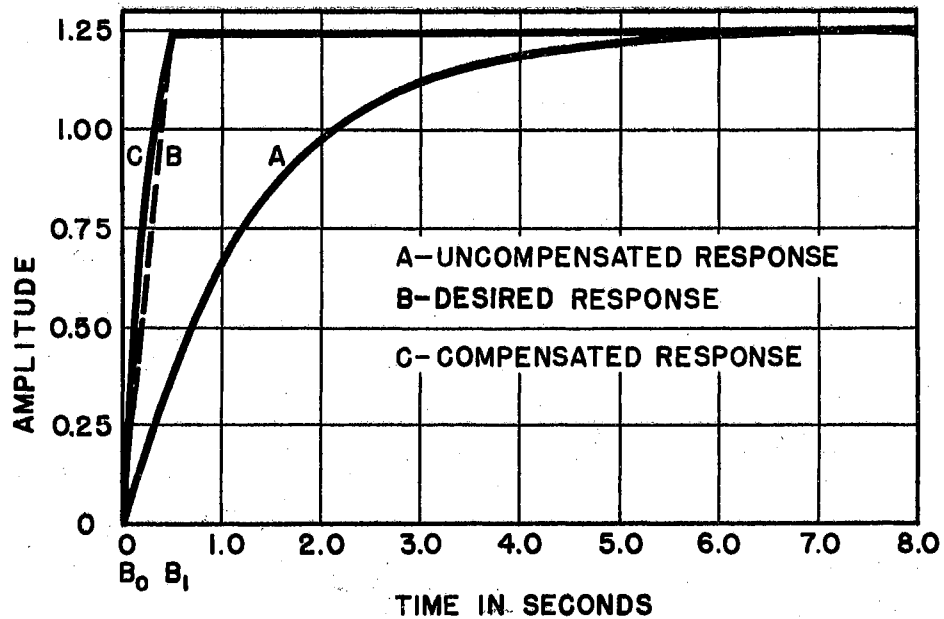


Fig. 9 Response Curves of First Order System With Unit Step Input

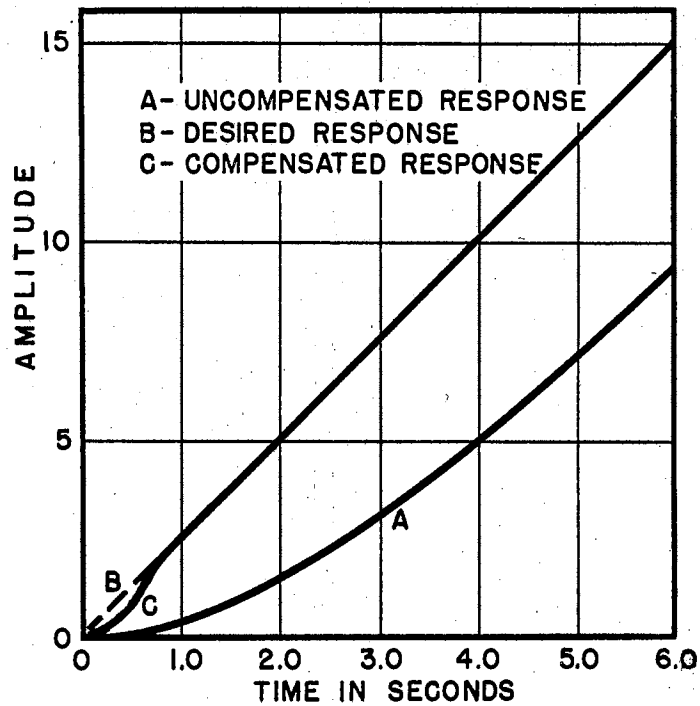


Fig. 10 Response Curves of First Order System With Ramp Input

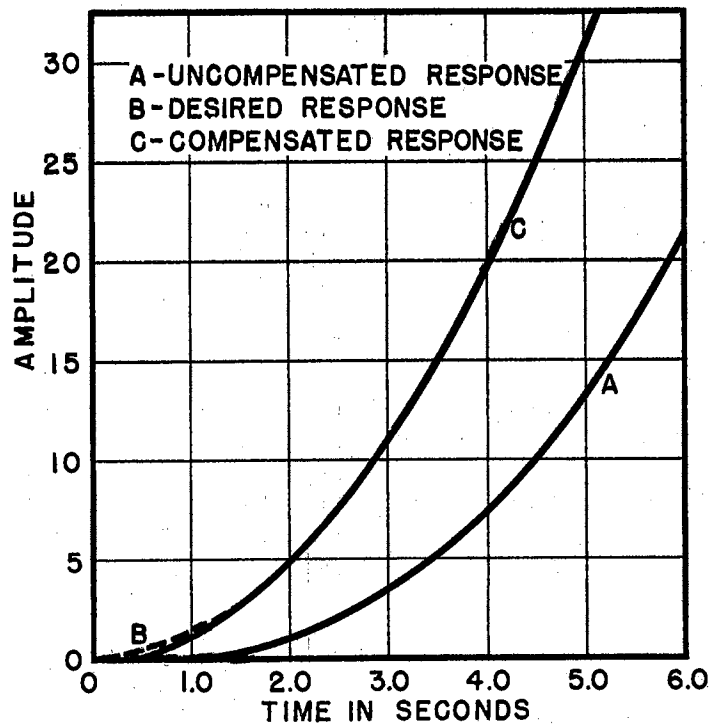


Fig. 11 Response Curves of First Order System With $t^2/2!$ Input

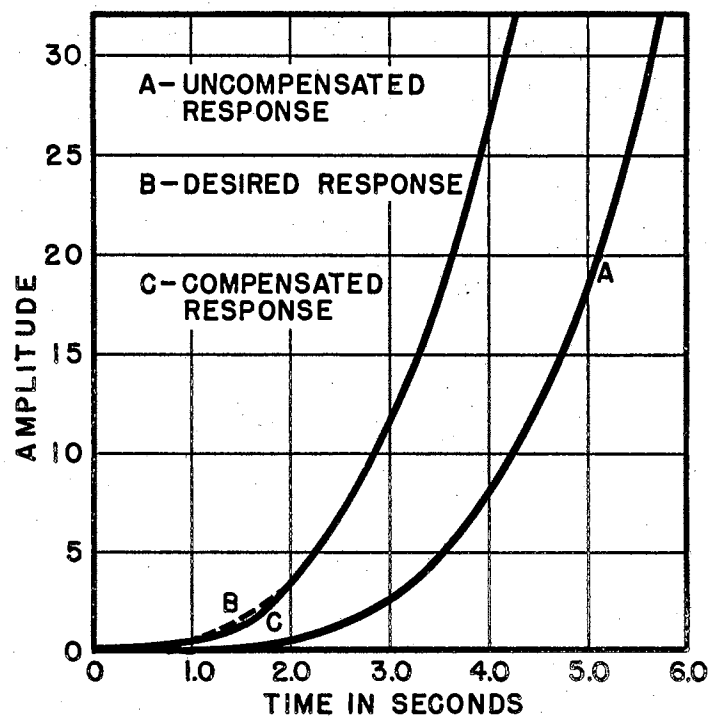


Fig. 12 Response Curves of First Order System With $t^3/3!$ Input

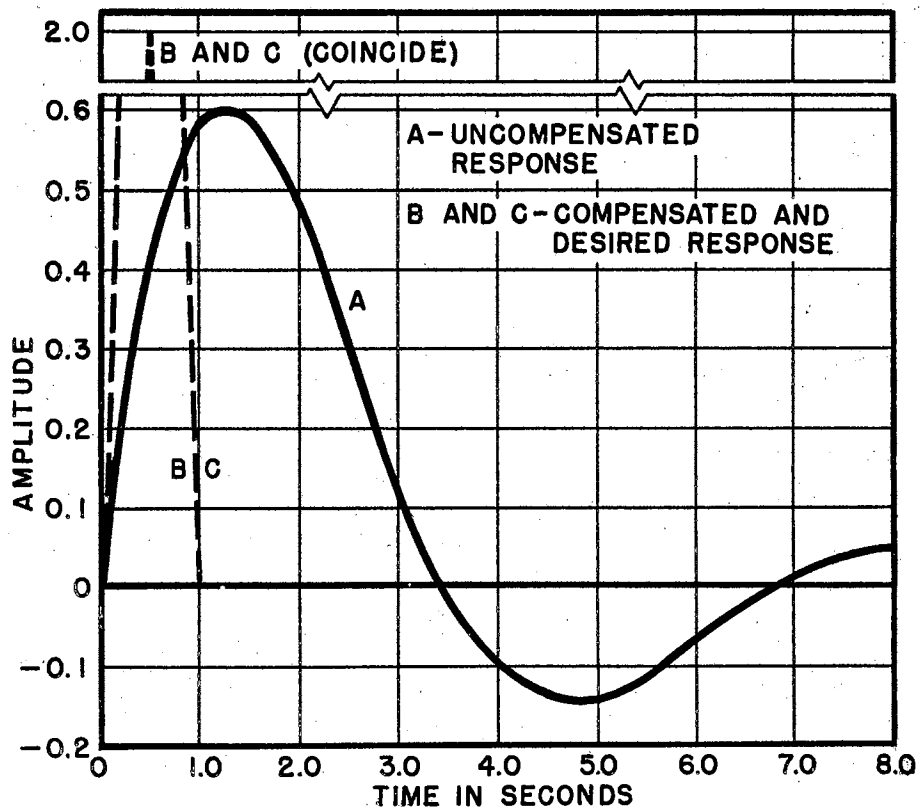


Figure 13 Response Curves of Second Order System With Unit Impulse Input

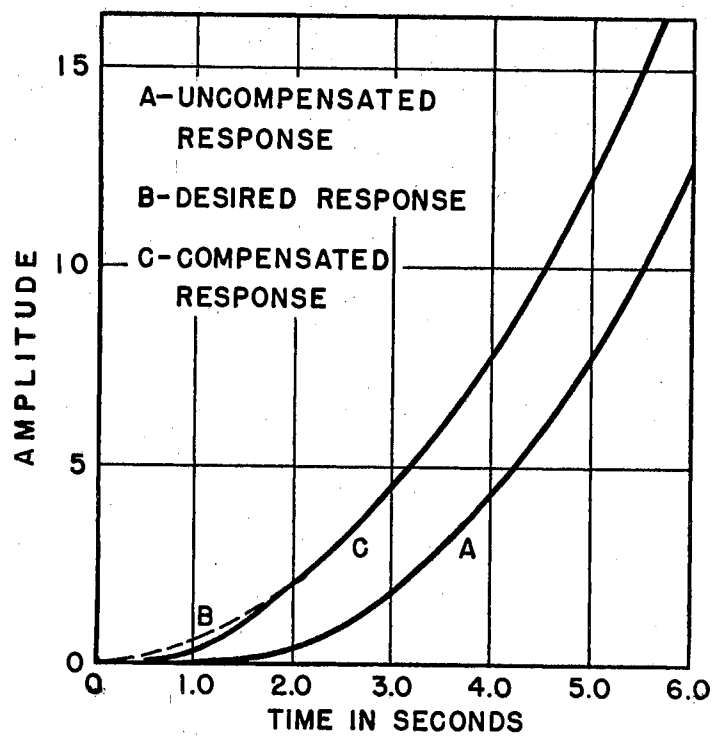


Fig. 14 Response Curves of Second Order System with $t^2/2!$ Input

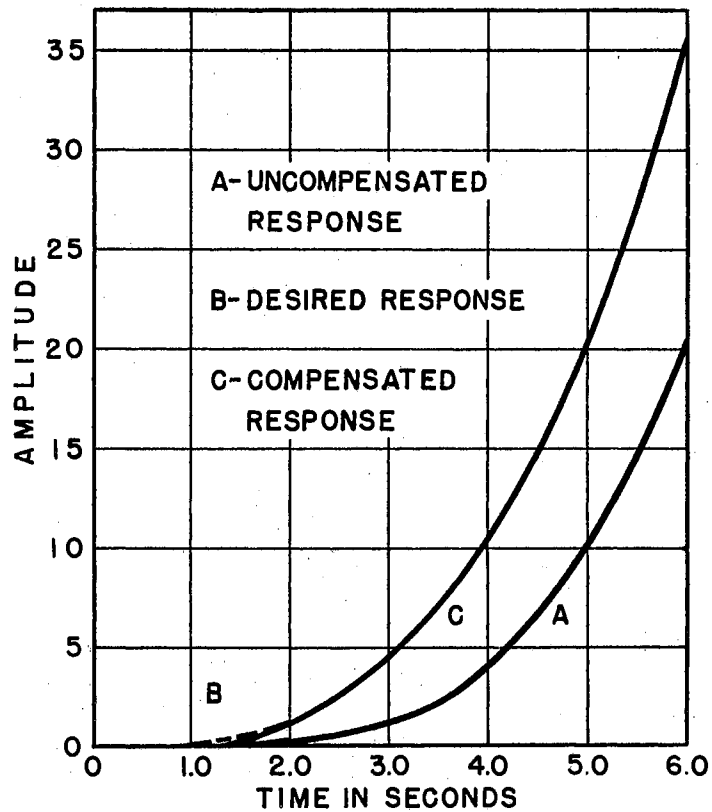


Fig. 15 Response Curves of Second Order System with $t^3/3!$ Input

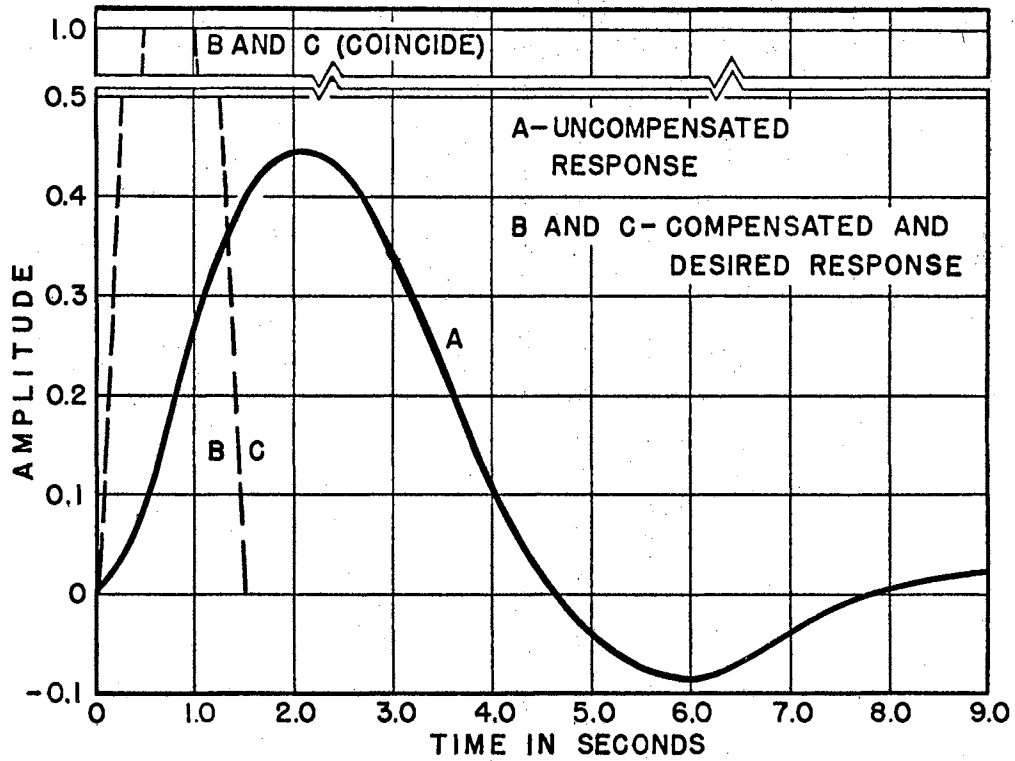


Fig. 16 Response Curves of Third Order System With Unit Impulse Input

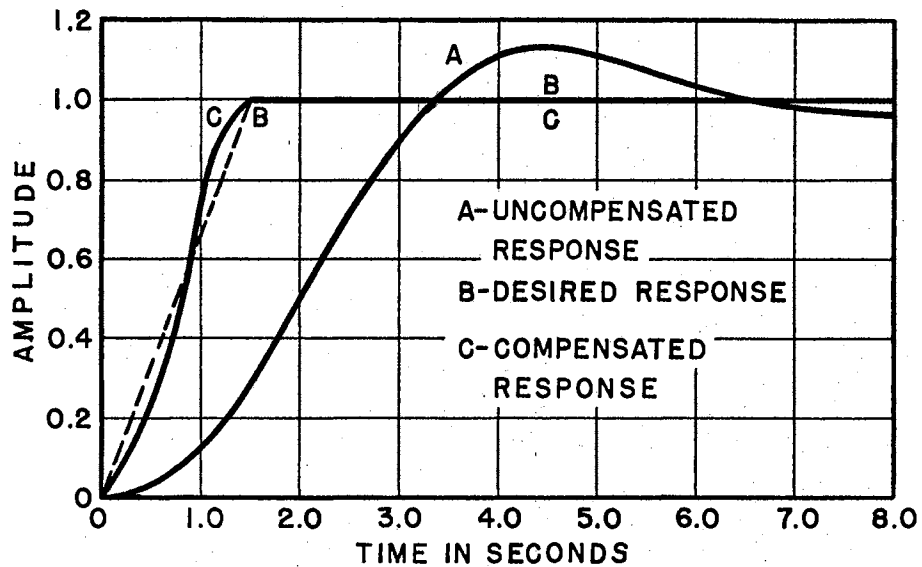


Fig. 17 Response Curves of Third Order System With Unit Step Input

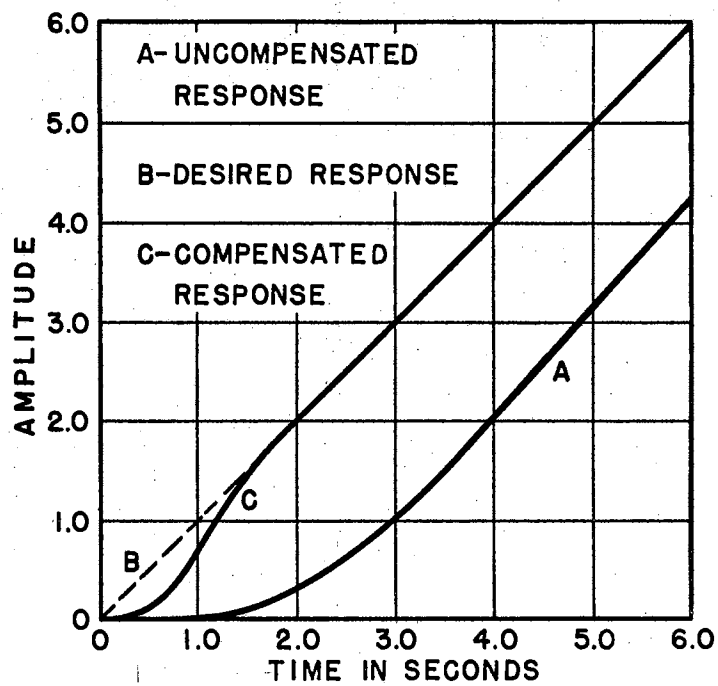


Fig. 18 Response Curves of Third Order System With Ramp Input

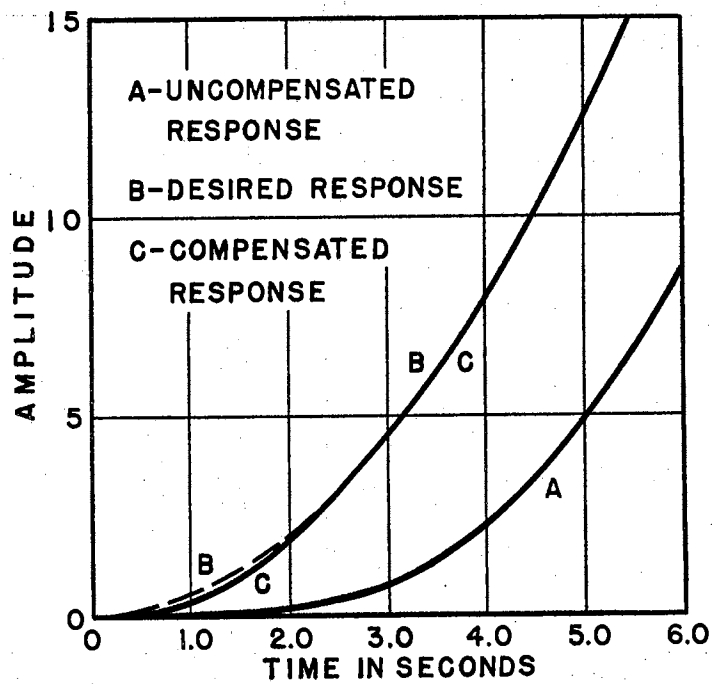


Fig. 19 Response Curves of Third Order System With $t^2/2!$ Input

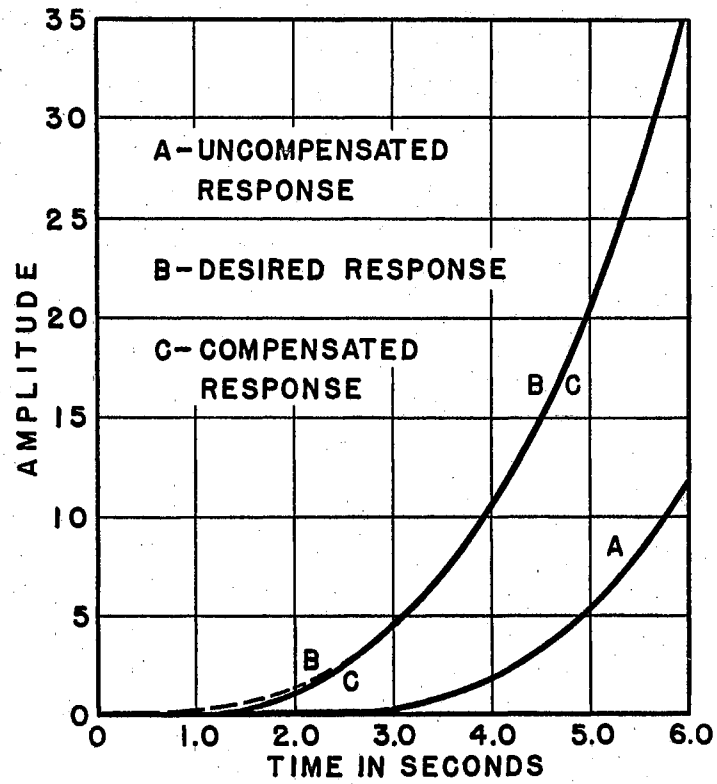


Fig. 20 Response Curves of Third Order System With $t^3/3!$ Input

CHAPTER III

CONCLUSIONS AND DISCUSSIONS

PART I

CONCLUSION

Surveying the result of the synthesis, some generalized properties of the principles and process of the design are summarized as follows:

(1) If a physical system can be characterized by a linear differential equation of k th order, and if a command signal is a function of t of the order of n , then the delay line compensator employing $(k + n + 1)$ taps can fulfill the criteria upon proper adjustment. If a first order impulse is used, the number of taps should be the same as the unit step function. Further, this result is independent of the magnitude of the command function.

(2) The delay line device can force the output to follow or match the input within a small time interval. This time interval actually equals the delay provided by the delay line. Thus we may minimize the delay length by using higher amplifier gains; and also we can use lower gains by using a longer delay line.

(3) The delay device can force the response to follow or match the command signal for all times greater than or equal to some fixed time interval.

(4) The length of the delay line is determined by the number of taps required and the delay length between taps. The total length should be:

$$L = v \cdot (k + n) \quad (3-1)$$

Where L is the total length of the line in seconds. v is the delay length between two taps in seconds. K and n are positive integers as defined in (1).

(5) For a given system and a given command function, the minimum number of taps (or the minimum number of amplifiers) is determined as stated above in (1). If the length of the delay line is changed, the gain of the amplifiers must be changed. The behaviour is that for the longer delay line, smaller amplifier gain factors may be used.

(6) A common characteristic among the compensators is that the polarities of these amplifiers take a sign opposite to those adjacent. (The few exceptional cases will be discussed later).

(7) This synthesis was carried up to a third order system, but the process showed that theoretically one can design the compensator, using this method, for any higher order of systems with the driving function of this kind up to any degree.

PART II

DISCUSSIONS

(1) Something About the Design:

The Time-Series Method for the design of delay-line compensators is an improvement over earlier methods. The advantages of this method are:

- (a) Data can be used directly from graphical records without ever having to find a mathematical expression of the output. Or, if tabulated data is available, it can be used directly.
- (b) On the other hand, if the transfer function is available, the output data is not required.
- (c) All mathematical operations are algebraic. Complex analysis is not required.
- (d) The effects of adding sampling points or of shifting sampling points are readily observed since all the analysis is in the time domain.
- (e) Compensation of this kind is perfectly smooth with no overshoot and no oscillation. The compensator can force the output to follow the input quickly within a very short time interval.
- (f) The accuracy of this method can be held to the range of 1% to 0.01% upon proper calculation. (The problems related to accuracy will be discussed later).

(2) Design of the Compensator With Slide-Rule Accuracy:

Generally speaking, in this study two points of view are involved.

The first is theoretical proof of the applicability of this method. The result is quite satisfactory in this regard. The second question is, "Will this design method work on some actual problems with slide-rule accuracy?"

In answering this question we made a little further study under the following considerations:

- (a) Take the data on the graph.
- (b) Assume the desired response directly, without making any modifications.
- (c) Calculate the values with a slide-rule.

Finally, we find that the process is limited by the accuracy of the prescribed desired response. The results of this analysis are:

(a) For impulse responses, the procedure and the results are almost the same. Even with the use of the slide-rule, the answer is still accurate enough.

(b) For step input in the low order systems (say first and second order) this work gives quite close results. As for third order systems, due to the difference between the prescribed desired response and the assumed value, a small deviation occurred in the compensated system which can be found in Curve C of Fig. 21. The system used here is the same as Fig. 17.

(c) As for higher order system and higher order input, the process is very difficult. Some modifications are needed. However, if the necessary modification is made, slide-rule calculation can work well.

(3) Accuracy of the Method:

The problem related to the accuracy of the design method is mainly

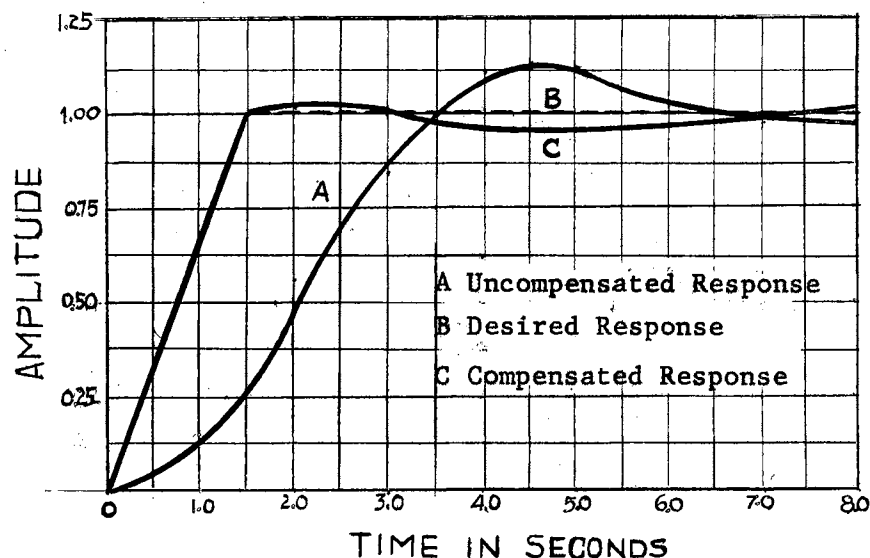


Fig. 21. Third Order System Step Command
Designed by Slide Rule Accuracy

dependent upon the prescribed desired response. If high accuracy is required, a precise calculation is needed.

However, in practice with the time series division, one finds that the accuracy is greatly affected by the exactness of the first few terms of these two time series. This effect was pointed out by W. H. Huggins (18) several years ago. The difficulty arises because the long division process is controlled by the leading term in the series. When this leading term is small, even a small error is enough to produce an appreciable error component in the quotient. When this quotient is used in the division process, large error will be introduced into the remainders after the first subtraction. In subsequent steps these errors will be repeatedly amplified and propagated. It is apparent, when the subsequent error is large enough, the division operation will not lead to a quotient series that converges.

The error occurs in several ways such as:

- (a) Determination of constants in solving the time responses.
- (b) Error introduced in taking the data from a graph.
- (c) The error introduced in the estimated response.
- (d) The error introduced by the elimination of the effective value of the lower digits.

So far as this design method is concerned, the influence of the above effect might be overcome. The reason is that the leading term of the desired response is determined by the uncompensated response and some known quantities. This can be seen in the Amplitude Constraint Method. Since the attention is focused on the leading term of the uncompensated response, one can control the error by carefully calculating the few leading terms of the uncompensated response. Of course a better design will be yielded when the elimination of other possible errors is attained.

(4) Effect of Elimination of the Amplifier.

It will be observed that a few irregular amplification factors appear in the list of Table VII - VIII. These values have two common features:

- (a) Negligible magnitude.
- (b) Irregular polarity.

Those data are exceptions of the statement of the conclusion in Part I of this Chapter. (6). The possibility of the elimination of those irregularities will now be considered.

First, consider the variation of the amplifier gains. When the amplifier gain is equal to unity, it is merely connected directly. When the gain is quite small, we might consider it an open connection.

Secondly, consider the effect of the elimination of amplifier. From

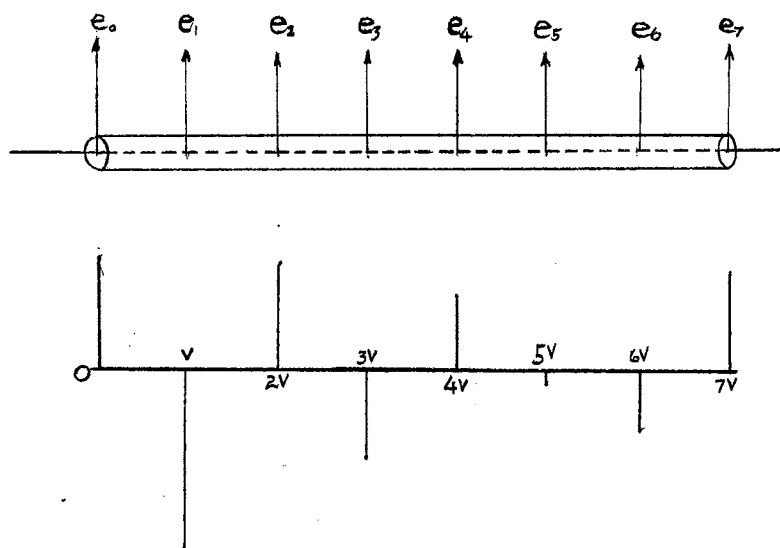


Fig. 22 The Standing Pulse Along The Delay Line

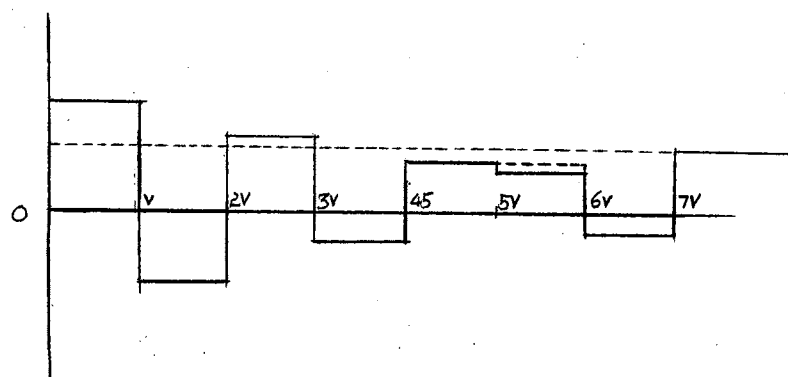


Fig. 23 Step Compensation of Delay Line Compensator

Fig. 23 we can find, if B_5 is absent the step compensation curve at this time interval changes to the dotted line. The result will be the same. From the above two points of view, it is possible to omit those which are insignificant.

PART III

RECOMMENDATIONS FOR FUTURE STUDY

It would appear that the future study might continue along one of the following directions:

(A) Synthesizing the delay line compensators using time series method, for the other type of command functions such as sinusoidal or exponential command, etc. It can be expected that the time series method will be useful in such design.

(B) The second proposal is to design the delay line compensator using time series method for composite command signal. That is, the signal applied to a given system is a linear combination of more than one simple function.

Another similar object is to design the compensator using continuous varied command function which is considered to be piece-wise continuous as shown below:

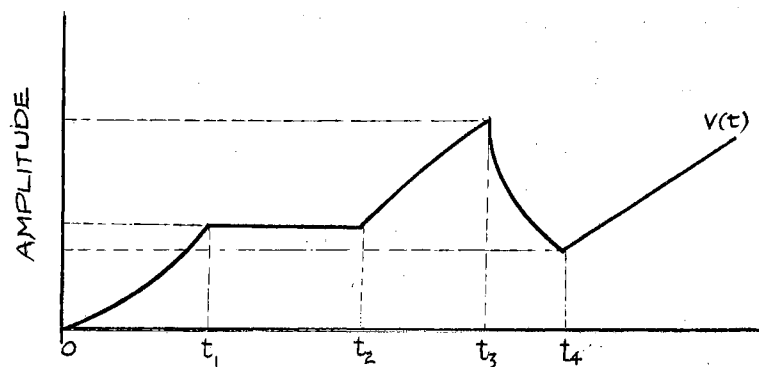


Fig. 24 Varied Continuous Command Function

(C) The design of delay line compensators in this paper is based on the assumption that the delay lines used have equi-distant taps. Work might be extended along another line, "What is the effect of variation of tap spacing?"

The behaviour with variation of length between taps has been discussed by Thompson. (16). The result is that for a given system with a given command signal, a minimum number of taps (equal space between taps) are determined. The variation of delay length between taps affects only the gains of the amplifiers in his example. It is suggested that study be given to the position of taps to get optimum results.

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APPENDIX A

ALGEBRA OF TIME SERIES

The purpose of this appendix is to illustrate some operational methods of the time series. First let us consider how a time function is expressed by using a time series.

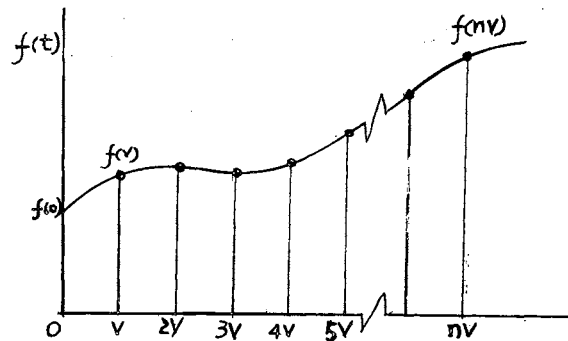


Fig. A - 1 Sampled-data Time Function

A given function $f(t)$ as shown in Fig. A-1 having values $f(nv)$ for t equal nv where the n are integers and the v are arbitrary small increments of t can be expressed approximately as the sequence of function $f(nv)$:

$$f(t) = f(0), f(v), f(2v), \dots, f(nv), \dots \quad (\text{A-1})$$

For a convenient notation which leads to operational methods let the sequence be written as a sum:

$$f(t) \approx \sum_{n=0}^{\infty} f(nv)x^n \quad (\text{A-2})$$

where x^n is the time correspondence which indicates the delay of each pulse at time equal nv . Finally the form of a time series is:

$$f(x) = f(0) + f(v)x + f(2v)x^2 + \dots + f(nv)x^n \quad (A-3)$$

This notation is exactly the same as the P-Transform. (6). The time series of this form has its great advantage in that it can be manipulated as a polynomial. The following several paragraphs will introduce such good properties.

(1) Addition and Subtraction:

It is evident from geometrical considerations that the sum of two time series is found by adding their values at each instant. For example,

$$\begin{aligned} f_1(x) &= a_0 + a_1x + a_2x^2 + \dots \\ \text{and } f_2(x) &= b_0 + b_1x + b_2x^2 + \dots \end{aligned} \quad (A-4)$$

the the sum of the two functions is,

$$f_1(x) + f_2(x) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \dots \quad (A-5)$$

Subtraction is the reverse operation of addition.

(2) Multiplication and Division:

Multiplication and division of time series are performed in exactly the same way as multiplication and division of polynomials:

$$\begin{aligned} f_1(x) &= a + bx + cx^2 \\ f_2(x) &= A + Bx + Cx^2 \end{aligned} \quad (A-6)$$

the product of $f_1(x)$ and $f_2(x)$:

$$F(x) = f_1(x) \cdot f_2(x)$$

$$\begin{array}{r} a + bx + cx^2 \\ A + Bx + Cx^2 \\ \hline \end{array}$$

$$\begin{array}{r} Aa + Abx + Acx^2 \\ Bax + Bbx^2 + Bcx^3 \\ Cax^2 + Cbx^3 + Ccx^4 \\ \hline \end{array}$$

$$F(x) = Aa + (Ab + Ba)x + (Ac + Bb + Ca)x^2 + (Bc + Cb)x^3 + Ccx^4 \quad (A-7)$$

And for division we have,

$$\frac{F(x)}{f_1(x)} = f_2(x)$$

$$\begin{array}{r}
 \begin{array}{r}
 A + Bx + Cx^2 \\
 \hline
 Aa + (Ab + Ba)x + (Ac + Bb + Ca)x^2 + (Bc + Cb)x^3 + Ccx^4 \\
 \hline
 Aa + Abx + Acx^2 \\
 \hline
 Bax + (Bb + Ca)x^2 + (Bc + Cb)x^3 + Ccx^4 \\
 \hline
 Bax + Bbx^2 + Bcx^3 \\
 \hline
 Cax^2 + Cbx^3 + Ccx^4 \\
 \hline
 Cax^2 + Cbx^3 + Ccx^4
 \end{array}
 \end{array}$$

It is shown clearly here how the notation used facilitates the manipulations by keeping the sequences of increments properly labeled throughout the operation. This is emphasized by using more simplified notation in the following example of multiplication:

$$f_1(x) = x^3 + x^4 + x^5$$

$$f_2(x) = x + 2x^2 + x^3$$

We write them in an alternate manner,

$$f_1(t) = 0, 0, 0, 1, 1, 1,$$

$$f_2(t) = 0, 1, 2, 1$$

(A-8)

By the polynomial multiplication as performed below:

$$\begin{array}{r}
 0, 0, 0, 1, 1, 1 \\
 \underline{0, 1, 2, 1} \\
 0, 0, 0, 0, 1, 1, 1 \\
 0, 0, 0, 0, 2, 2, 2 \\
 \underline{0, 0, 0, 1, 1, 1} \\
 0, 0, 0, 0, 1, 3, 4, 3, 1
 \end{array}$$

$$f_1(x) \cdot f_2(x) = x^4 + 3x^5 + 4x^6 + 3x^7 + x^8$$

(A-9)

A geometrical interpretation of the above example as shown in the following:

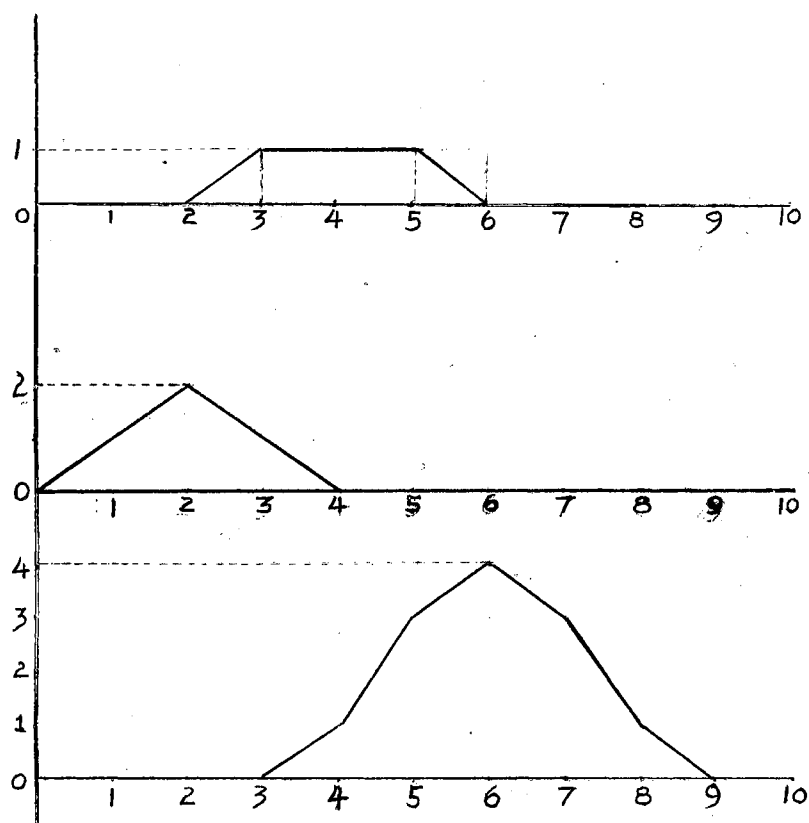


Fig. A - 2 Delay of Pulse as a Result of Time Series Multiplication

APPENDIX B

THE DELAY OPERATOR*

The Delay Operator used in the time domain analysis is:

$$e^{-Tp} \quad (B-1)$$

where T is the delay time in second, p represents the operation of the differentiation. The delay operator acting upon F(t) has the significant

$$e^{-Tp} F(t) = F(t - T) \quad (B-2)$$

The validity of relationship can be demonstrated by expanding e^{-Tp} in power series of p and comparing that of F(t - T) by Taylor's Theorem:

$$\begin{aligned} e^{-Tp} F(t) &= [1 + Tp + T^2 p^2 / 2! + \dots] F(t) \\ &= F(t) + TF'(t) + \frac{T^2}{2!} F''(t) + \dots \\ &= F(t - T) \end{aligned} \quad (B-3)$$

The Laplace Transformation of a delayed function is precisely Eq. (B-2), if the complex variable of the Laplace Transform has the same meaning as the differentiation factor p, the transform of F(t) is:

$$L [F(t)] = \int_0^{\infty} F(t) e^{-st} dt = F(s) \quad (B-4)$$

If F(t - T) is substituted in Eq. (B-4).

$$\int_0^{\infty} F(t - T) e^{-s(t-T)} dt = e^{-sT} \int_0^{\infty} F(t-T) e^{-st} dt \quad (B-5)$$

It follows that F(t) = 0 at 0 < t < T then

$$\int_0^{\infty} F(t-T) e^{-st} dt = e^{-sT} F(s) \quad (B-6)$$

This latter condition is always satisfied in the development.

* Adapted from reference (3).

APPENDIX C

MEANS OF FINDING TIME RESPONSE IN THE FORM OF TIME SERIES

In this appendix, three methods of finding time response in the form of a time series will be introduced. (1) Use the method of Boxer and Thaler to obtain time series directly. (2) Use the formal Laplace Transform method of getting the solution of time response, tabulate the data, write the time series. (3) Take the data from a graph when this response is a graphical solution.

(1) Method of Boxer and Thaler:

The step to obtain the time series when the system overall transfer function is known is:

- (a) Express the function $F(s)$ as a rational fraction in power of s by dividing the numerator and denominator by s^m .
- (b) Substitute for s a rational fraction in power of z^{-1} obtained from Z-transform table and rearrange $F(s)$ as a rational fraction in power of z^{-1} .
- (c) Divide the resulting expression by T where T is the time interval between points at which the solution is desired.
- (d) Expand the fraction by synthetic division into a series of the form:

$$D_0 + D_1 z^{-1} + D_2 z^{-2} + D_3 z^{-3} + \text{-----} + D_n z^{-n}$$

where D , the coefficient of z , is the approximate value of the time response at $t=nT$. Change the expression to the following:

$$D_0 + D_1x + D_2x^2 + \text{-----} + D_nx^n + \text{-----}$$

Example 1--Second order system step input:

This example will be based upon a second order system as shown in Fig. C-1. The Laplace transform of the output is given by:

$$Q(s) = \frac{1}{s^3 + s^2 + s}$$

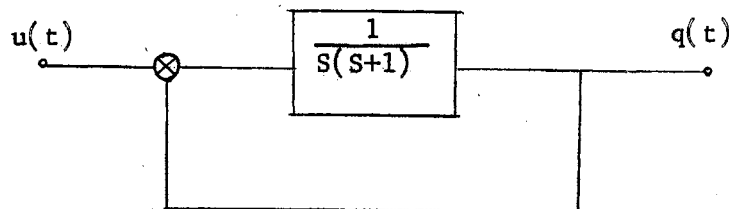


Fig. C-1 Third Order Control System

Following the step-by-step procedure outlined above, the transform is expressed in the power of S :

$$Q(s) = \frac{s^{-3}}{1 + s^{-1} + s^{-2}}$$

substituting the corresponding forms from Table IX and dividing the result by T ,

$$Q(s) = \frac{6T (z^{-1} + z^{-2})}{(12+6T+T^2) - (36 + 6T-9T^2) z^{-1} + (36-6T-9T^2)z^{-2} - (12-6T+T^2)z^{-3}}$$

the solution is obtained by choosing T and dividing the denominator into the numerator. Suppose we choose $T = 0.5$ and here we use x instead of z which causes the output to be expressed as:

TABLE IX
Z - TRANSFORM

s^{-k}	Z - Transform $F_k(z^{-1})$
s^{-1}	$\frac{T}{2} \frac{1 + z^{-1}}{1 - z^{-1}}$
s^{-2}	$\frac{T^2}{12} \frac{1 + 10z^{-1} + z^{-1}}{(1 - z^{-1})^2}$
s^{-3}	$\frac{T^3}{2} \frac{z^{-1} + z^{-2}}{(z^{-1} - z^{-1})^3}$
s^{-4}	$\frac{T^4}{6} \frac{(z^{-1} + 4z^{-2} + z^{-3})}{(1 - z^{-1})^4} = \frac{T^4}{720}$
s^{-5}	$\frac{T^5}{24} \frac{z^{-1} + 11z^{-2} + 11z^{-3} + z^{-4}}{(1 - z^{-1})^5}$

$$Q(s) = \frac{1.5s + 1.5s^2}{15.25 - 36.75s + 30.75s^2 - 9.25s^3}$$

Carrying out the long division process,

$$15.25 - 36.75s + 30.75s^2 - 9.25s^3 \overline{) 0.0984s + 0.335s^2 + 0.610s^3 + .853s^4}$$

The points obtained in this case are plotted on the figure:

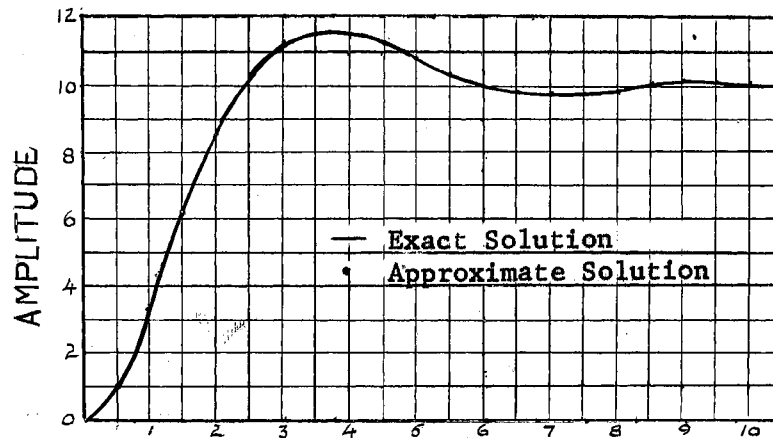


Fig. C-2 Exact and Approximate Solution To Third-Order System

(II) Laplace Transform method:

If the system transfer function is given, take inverse Laplace Transform to get the time function of the response through some necessary operations. Once the time function is found, calculate the response point-by-point by substitution of t into the time function. These values calculated are the coefficients of the time series.

Example 2:

Suppose the overall transfer function of a third order system with step input is:

$$Q(s) = \frac{1}{s(s+1)(s+0.8s+1)}$$

by Heaviside's expansion theorem we get:

$$Q(s) = \frac{1}{s} - \frac{5}{6(s+1)} - \frac{1}{6} \frac{s + 5.8}{(s + 0.4) + (0.9165)^2}$$

using Laplace Transform table find the solution

$$Q(t) = 1 - \frac{5e^{-t}}{6} - 0.996e^{-0.4t} \sin(0.9165t + 0.168)$$

Calculate the time response point-by-point using the time interval

0.5 second. The values as listed below:

t	Q(t)	t	Q(t)
0.0	0.000000	5.5	1.090705
0.5	0.030905	6.0	1.048565
1.0	0.120914	6.5	1.010015
1.5	0.284099	7.0	0.981849
2.0	0.492739	7.5	0.966447
2.5	0.707535	8.0	0.962781
3.0	0.893903	8.5	0.967767
3.5	1.030150	9.0	0.977573
4.0	1.109276	9.5	0.988647
4.5	1.136551	10.0	0.998346
5.0	1.125026		

The time series of this time function will be:

$$Q(x) = 0.031x + 0.121x^2 + 0.284x^3 + \text{-----} \text{ etc.}$$

(III) Graphical Data:

In this case, this is the great advantage of the time series approach. The data taken from oscillographic or other graphical records does not require the operation of finding an exact mathematical expression for use in the calculation. The time series representation of such data may be written down by inspection. The values of the function at the successive equidistant points become the coefficients of the x in the time series.

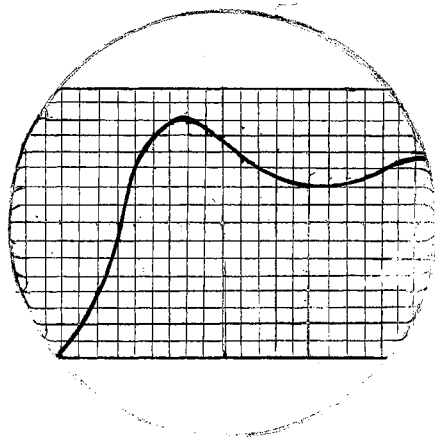


Fig. C-3

An Oscilloscopic
Graph

From the graph, taken directly from the oscilloscope, we can write the time series of such a response as:

$$Q(x) = 0.005 + 0.02x + 0.04x^2 + 0.07x^3 + 0.11x^4 + \text{-----}$$

APPENDIX D

LIST OF DATA

This appendix contains three groups of data. The nature of each group of data is briefly specified as follows:

(1) Group I

The functions listed in Tables X-XII are the solutions of uncompensated time responses for various types of systems (as listed in Table I) with various types of command functions (as listed in Table II). All those functions are solved by using Inverse Laplace-Transformation.

(2) Group II

All data listed in this group are the calculated responses of each individual case appearing in the synthesis. For each table three kinds of responses are contained.

- (a) Uncompensated response is calculated by substituting t in the equations listed in Group I. Those are the exact solutions of the uncompensated systems.
- (b) The desired response is assigned based on the design criteria.
- (c) The compensated response is the response of the compensated system designed by Time-Series method.

All curves plotted in Chapter II, as a result of Time-Series design method are based upon the above tabulated data.

(3) Group III

The data contained in this group are the results designed by slide-rule accuracy referring to Part II-C, Chapter III.

TABLE X

SOLUTIONS OF TIME RESPONSE OF FIRST ORDER SYSTEM

Command Function	Overall System Transfer Function	Uncompensated System Response
$\delta(t)$	$\frac{1}{s + 0.4}$	$R(t) = e^{-0.4t}$
$u(t)$	$\frac{1}{s(s + 0.4)}$	$R(t) = 2.5(1 - e^{-0.4t})$
$t \cdot u(t)$	$\frac{1}{s^2(s + 0.4)}$	$R(t) = 2.5t - 6.25(1 - e^{-0.4t})$
$\frac{t^2}{2!} \cdot u(t)$	$\frac{1}{s^3(s + 0.4)}$	$R(t) = 1.25t^2 - 6.25t + 15.625(1 - e^{-0.4t})$
$\frac{t^3}{3!} \cdot u(t)$	$\frac{1}{s^4(s + 0.4)}$	$R(t) = \frac{2.5}{3!}t^3 - (2.5)^2 \frac{t^2}{2!} + (2.5)^3 - (2.5)^4(1 - e^{-0.4t})$

TABLE XI

SOLUTIONS OF TIME RESPONSE OF SECOND ORDER SYSTEM

Command	Overall Transfer Function	Uncompensated System Response
$\delta(t)$	$\frac{1}{s^2 + 0.8s + 1}$	$R(t) = K e^{-\alpha t} \sin \beta t$
$u(t)$	$\frac{1}{s(s^2 + 0.8s + 1)}$	$R(t) = 1 - K e^{-\alpha t} \sin (\beta t + \theta_1)$
$t \cdot u(t)$	$\frac{1}{s^2(s^2 + 0.8s + 1)}$	$R(t) = t - 2a + K e^{-\alpha t} \sin (\beta t + \theta_2)$
$\frac{t^2}{2!} u(t)$	$\frac{1}{s^3(s^2 + 0.8s + 1)}$	$R(t) = \frac{t^2}{2!} - 2at - b + K e^{-\alpha t} \sin (\beta t + \theta_3)$
$\frac{t^3}{3!} u(t)$	$\frac{1}{s^4(s^2 + 0.8s + 1)}$	$R(t) = \frac{t^3}{3!} - at^2 - bt + c - K e^{-\alpha t} \sin (\beta t + \theta_4)$

NOTE:

$a = 0.400000$
 $b = 0.360000$
 $c = 1.088000$
 $K = 1.091100$
 $\alpha = 0.40000$

$\beta = 0.916515$
 $\theta_1 = 1.138120$
 $\theta_2 = 2.320000$
 $\theta_3 = 0.336266$
 $\theta_4 = 1.495500$

TABLE XII
SOLUTIONS OF TIME RESPONSE OF THIRD ORDER SYSTEM

Command Function	Overall System Transfer Function	Uncompensated System Response
$\delta(t)$	$\frac{1}{(s+1)(s^2 + 0.8s + 1)}$	$R(t) = K_1 e^{-t} - K_2 e^{-\alpha t} \sin(\beta t + \theta_0)$
$u(t)$	$\frac{1}{s(s+1)(s^2 + 0.8s + 1)}$	$R(t) = 1 - K_1 e^{-t} - K_2 e^{-\alpha t} \sin(\beta t + \theta_1)$
$t \cdot u(t)$	$\frac{1}{s^2(s+1)(s^2 + 0.8s + 1)}$	$R(t) = t - a + K_1 e^{-t} + K_2 e^{-\alpha t} \sin(\beta t + \theta_2)$
$\frac{t^2}{2!} \cdot u(t)$	$\frac{1}{s^3(s+1)(s^2 + 0.8s + 1)}$	$R(t) = \frac{t^2}{2!} - at + b - K_1 e^{-t} - K_2 e^{-\alpha t} \sin(\beta t + \theta_3)$
$\frac{t^3}{3!} \cdot u(t)$	$\frac{1}{s^4(s+1)(s^2 + 0.8s + 1)}$	$R(t) = \frac{t^3}{3!} - \frac{a}{2} t^2 + bt - c + K_1 e^{-t} + K_2 e^{-\alpha t} \sin(\beta t + \theta_4)$

Note:

$a = 1.800000$	$K_2 = 0.996027$	$\theta_1 = 0.168133$
$b = 1.440000$	$\alpha = 0.400000$	$\theta_2 = 1.327620$
$c = 0.352000$	$\beta = 0.916515$	$\theta_3 = 2.486587$
$K_1 = 0.833333$	$\theta_0 = 2.150540$	$\theta_4 = 0.504400$

TABLE XIII

RESPONSES OF FIRST ORDER SYSTEM
WITH UNIT IMPULSE INPUT

Time (Sec.)	Uncompensated Response	Compensated Response
0.0	1.000000	0.000000
0.1	0.960789	5.867555
0.2	0.923116	5.637484
0.3	0.886920	5.416436
0.4	0.852144	5.204054
0.5	0.818731	5.000000
0.6	0.786628	0.000000
0.7	0.755784	0.000000
0.8	0.726149	0.000000
0.9	0.697676	0.000000
1.0	0.670320	0.000000
1.1	0.644036	0.000000
1.2	0.618783	0.000000
1.3	0.594520	0.000000
1.4	0.571209	0.000000
1.5	0.548811	0.000000
2.0	0.449329	0.000000
2.5	0.367879	0.000000
3.0	0.301194	0.000000
3.5	0.246597	0.000000
4.0	0.201897	0.000000
4.5	0.165299	0.000000
5.0	0.135335	0.000000
5.5	0.110803	0.000000
6.0	0.090717	0.000000
6.5	0.074274	0.000000
7.0	0.060810	0.000000
7.5	0.049787	0.000000
8.0	0.040762	0.000000
8.5	0.033373	0.000000
9.0	0.027324	0.000000
9.5	0.022371	0.000000
10.0	0.018315	0.000000

TABLE XIV
RESPONSES OF FIRST ORDER SYSTEM
WITH UNIT STEP INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	1.250000	0.000000	0.000000
0.1		0.096120	0.174554
0.2		0.184810	0.335615
0.3		0.266667	0.484273
0.4		0.342350	0.621708
0.5		0.412090	0.748355
0.6	1.250000	0.476540	0.865397
0.7		0.536000	0.973376
0.8		0.590880	1.073038
0.9		0.641550	1.165055
1.0		0.688320	1.249989
1.1	1.250000	0.729570	1.246445
1.2		0.771387	1.250903
1.3		0.808170	1.250035
1.4		0.842150	1.250009
1.5		0.873500	1.246465
1.6	1.250000	0.902450	1.250019
1.7		0.929170	1.249999
1.8		0.953840	1.249020
1.9		0.976610	1.250015
2.0		0.997620	1.250000
2.5	1.250000	1.079135	1.247900
3.0		1.136610	1.227685
3.5		1.173880	
4.0		1.199050	1.248300
4.5			
5.0	1.250000	1.227100	1.250100
5.5			
6.0		1.239720	1.250000
6.5			
7.0		1.245370	1.250000
7.5			
8.0		1.250000	1.250000
8.5			
9.0		1.250000	1.250000
9.5			
10.0		1.250000	1.250000

TABLE XV

RESPONSES OF FIRST ORDER SYSTEM
WITH RAMP INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.00000	0.000000	0.000000
0.1	0.25000	0.004934	0.040373
0.2	0.50000	0.019477	0.159381
0.3	0.75000	0.043253	0.353939
0.4	1.00000	0.075899	0.621087
0.5	1.25000	0.117067	0.957970
0.6	1.50000	0.166424	0.315657
0.7	1.75000	0.223648	1.647710
0.8	2.00000	0.288431	1.955149
0.9	2.25000	0.360477	2.238935
1.0	2.50000	0.439500	2.500000
1.1	2.75000	0.525228	2.749996
1.2	3.00000	0.617396	3.000002
1.3	3.25000	0.715753	3.249999
1.4	3.50000	0.820057	3.500003
1.5	3.75000	0.939973	3.750000
2.0	5.00000	1.558306	5.000000
2.5	6.25000	2.299246	6.249999
3.0	7.50000	3.132464	7.500000
3.5	8.75000	4.041231	8.750000
4.0	10.00000	5.011853	9.999999
4.5	11.25000	6.033118	11.250003
5.0	12.50000	7.095846	12.500000
5.5	13.75000	8.192519	13.749990
6.0	15.00000	9.316988	15.000016
6.5	16.25000	10.464210	16.249995
7.0	17.50000	11.630063	17.500003
7.5	18.75000	12.811169	18.749990
8.0	20.00000	14.004764	20.000010
8.5	21.25000	15.208583	21.249998
9.0	22.50000	16.420773	22.499992
9.5	23.75000	17.639818	23.750014
10.0	25.00000	18.864473	24.999992

TABLE XVI
RESPONSES OF FIRST ORDER SYSTEM
WITH $t^2/2$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
0.1	0.012500	0.000166	0.001645
0.2	0.050000	0.001308	0.012988
0.3	0.112500	0.004369	0.043388
0.4	0.200000	0.010253	0.101826
0.5	0.312500	0.019833	0.196954
0.6	0.450000	0.033939	0.334691
0.7	0.612500	0.053380	0.511435
0.8	0.800000	0.078922	0.721349
0.9	1.012500	0.111308	0.918873
1.0	1.250000	0.151250	1.218580
1.1	1.512500	0.199431	1.496614
1.2	1.800000	0.256509	1.793352
1.3	2.112500	0.323117	2.110553
1.4	2.450000	0.399858	2.449744
1.5	2.812500	0.487319	2.812500
2.0	5.000000	1.104234	4.999999
2.5	7.812500	2.064384	7.812500
3.0	11.250000	3.418841	11.249900
3.5	15.312500	5.209422	15.312291
4.0	20.000000	7.470367	20.000000
4.5	25.312500	10.229705	25.312000
5.0	31.250000	13.510386	31.249338
5.5	37.812500	17.331202	37.811657
6.0	45.000000	21.707531	44.998907
6.5	52.812500	26.651975	52.811250
7.0	61.250000	32.174842	61.248496
7.5	70.312500	38.284566	70.310790
8.0	80.000000	44.988091	79.998050
8.5	90.312500	52.291040	90.310270
9.0	101.250000	60.198070	101.247630
9.5	112.812500	68.712960	112.809790
10.0	125.000000	77.838820	124.997070

TABLE XVII

RESPONSES OF FIRST ORDER SYSTEM
WITH $t^{3/5}$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
.1	0.000417	0.000003	0.000028
.2	0.003333	0.000064	0.000682
.3	0.012500	0.000328	0.003510
.4	0.026667	0.001034	0.011060
.5	0.052083	0.002501	0.026758
.6	0.900000	0.005152	0.055072
.7	0.142917	0.009467	0.100216
.8	0.213333	0.016028	0.165953
.9	0.303750	0.025480	0.255225
1.0	0.416667	0.038542	0.370339
1.1	0.554583	0.056005	0.512701
1.2	0.720000	0.078727	0.683968
1.3	0.915417	0.107624	0.885535
1.4	1.143333	0.143688	1.119458
1.5	1.406250	0.187953	1.138771
1.6	1.706667	0.241526	1.693040
1.7	20.47083	0.305560	2.037625
1.8	2.430000	0.381258	2.424205
1.9	2.857917	0.469886	2.855225
2.0	3.333333	0.572747	3.333333
2.5	6.510417	1.349456	6.517460
3.0	11.250000	2.702898	11.257213
3.5	17.864583	4.841028	17.868447
4.0	26.666667	7.990749	26.666664
4.5	37.968750	12.394488	37.966849
5.0	52.083333	18.307368	52.083325
5.5	69.322917	25.994913	69.330277
6.0	90.000000	35.731170	90.021587
6.5	114.427000	47.797140	114.470600
7.0	142.916670	62.479570	142.991220
7.5	175.981250	80.069810	175.895600
8.0	213.333333	100.863100	213.498240
8.5	255.885420	125.157820	256.111450
9.0	303.750000	153.254939	304.047270
9.5	357.239580	185.457190	357.620250
10.0	416.666667	222.069620	417.141730

TABLE XVIII
RESPONSES OF SECOND ORDER SYSTEM
WITH UNIT IMPULSE INPUT

Time (Sec.)	Uncompensated Response	Compensated Response
0.0	0.00000	0.00000
0.1	0.09600	0.48585
0.2	0.18300	0.95145
0.3	0.26200	1.32596
0.4	0.33300	1.68529
0.5	0.39519	2.00000
0.6	0.44800	1.55382
0.7	0.49300	1.04720
0.8	0.53000	0.73509
0.9	0.55900	0.32889
1.0	0.58034	0.00000
1.1	0.59400	0.00000
1.2	0.60100	0.00000
1.3	0.60200	0.00000
1.4	0.59600	0.00000
1.5	0.58733	0.00000
1.6	0.57200	0.00000
1.7	0.55100	0.00000
1.8	0.52800	0.00000
1.9	0.50300	0.00000
2.0	0.47350	0.00000
2.1	0.44200	0.00000
2.2	0.40800	0.00000
2.3	0.37300	0.00000
2.4	0.33800	0.00000
2.5	0.30163	0.00000
3.0	0.12555	0.00000
3.5	-0.01780	0.00000
4.0	-0.11031	0.00000
4.5	-0.15007	0.00000
5.0	-0.14643	0.00000

TABLE XIX

RESPONSES OF SECOND ORDER SYSTEM WITH $t^2/2$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	.000000	0.000000	0.000000
.1	.005000	0.000022	0.000302
.2	.020000	0.000080	0.001128
.3	.055000	0.000335	0.004703
.4	.080000	0.001009	0.014160
.5	.125000	0.002400	0.033680
.6	.180000	0.004873	0.067643
.7	.245000	0.008849	0.121432
.8	.320000	0.014799	0.196266
.9	.405000	0.023236	0.291748
1.0	.500000	0.034704	0.405388
1.1	.605000	0.049772	0.533559
1.2	.720000	0.069028	0.670667
1.3	.845000	0.093071	0.814398
1.4	.980000	0.122501	0.963254
1.5	1.125000	0.157929	1.117062
1.6	1.280000	0.199920	1.276625
1.7	1.445000	0.249080	1.443900
1.8	1.620000	0.305963	1.619752
1.9	1.805000	0.371110	1.804964
2.0	2.000000	0.445037	1.999999
2.5	3.125000	0.962357	3.125000
3.0	4.500000	1.758317	4.500011
3.5	6.125000	2.859606	6.124933
4.0	8.000000	4.272950	8.000000
4.5	10.125000	5.990332	10.125000
5.0	12.500000	7.995474	12.500000
5.5	15.125000	10.269846	15.601000
6.0	18.000000	12.797143	18.034200
6.5	21.125000	15.565848	21.125472
7.0	24.400000	18.569973	24.500585
7.5	28.125000	21.808447	28.125753
8.0	32.000000	25.283712	32.000922
8.5	36.125000	29.000067	36.126010
9.0	40.500000	32.962196	40.501200
9.5	45.125000	37.174089	45.126213
10.0	50.000000	41.638469	50.000500

TABLE XX

RESPONSES OF SECOND ORDER SYSTEM WITH $t^{3/3}$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
.1	0.000167	0.000005	0.000016
.2	0.001333	0.000010	0.000032
.3	0.004500	0.000016	0.000087
.4	0.020833	0.000226	0.001047
.5	0.020833	0.000226	0.003631
.6	0.036000	0.000577	0.009227
.7	0.057167	0.001248	0.019948
.8	0.085333	0.002410	0.038478
.9	0.121500	0.004289	0.066159
1.0	0.166667	0.007158	0.105573
1.1	0.221833	0.011349	0.158392
1.2	0.288000	0.017252	0.225500
1.3	0.366167	0.025315	0.307029
1.4	0.457333	0.036047	0.404231
1.5	0.562500	0.050017	0.516735
1.6	0.682667	0.067853	0.644708
1.7	0.818833	0.090243	0.788567
1.8	0.972000	0.117930	0.949034
1.9	1.143167	0.151715	1.126401
2.0	1.333333	0.192449	1.321737
2.1	1.543500	0.241035	1.535998
2.2	1.774667	0.298424	1.770248
2.3	2.027833	0.365609	2.025592
2.4	2.304000	0.443626	2.303180
2.5	2.604167	0.533546	2.604167
3.0	4.500000	1.201393	4.500000
3.5	7.145833	2.342884	7.145837
4.0	10.666667	4.113105	10.666671
4.5	15.187500	6.666560	15.179461
5.0	20.833333	10.151409	20.797364
5.5	27.726167	14.706888	27.633462
6.0	36.000000	20.463375	35.802996
6.5	45.770233	27.544226	45.424919
7.0	57.166667	36.068425	56.621602
7.5	70.312500	46.153234	69.518472
8.0	85.333333	57.916325	84.242531
8.5	102.354170	71.477130	100.922410
9.0	121.500000	86.957360	119.685150
9.5	142.895830	104.480960	140.659390
10.0	166.666667	124.173540	163.970040

TABLE XXI

RESPONSES OF THIRD ORDER SYSTEM
WITH UNIT IMPULSE INPUT

Time (Sec.)	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000
0.1	0.004768	0.052262
0.2	0.017761	0.194690
0.3	0.037448	0.410491
0.4	0.062376	0.683749
0.5	0.091227	1.000001
0.6	0.122809	1.237983
0.7	0.156054	1.3075129
0.8	0.190014	1.232969
0.9	0.223854	1.038127
1.0	0.256847	0.744996
1.1	0.288370	0.454666
1.2	0.317898	0.244273
1.3	0.344995	0.104595
1.4	0.369314	0.026121
1.5	0.390585	0.000004
2.0	0.446629	0.000000
2.5	0.421490	0.000000
3.0	0.336217	0.000000
3.5	0.221296	0.000000
4.0	0.105723	0.000000
4.5	0.010628	0.000000
5.0	-0.053008	0.000000
5.5	-0.083603	0.000000
6.0	-0.086412	0.000000
6.5	-0.070354	0.000000
7.0	-0.045087	0.000000
7.5	-0.018867	0.000000
8.0	0.002628	0.000000
8.5	0.016574	0.000000
9.0	0.022620	0.000000
9.5	0.022132	0.000000
10.0	0.017354	0.000000

TABLE XXII

RESPONSES OF THIRD ORDER SYSTEM
WITH UNIT STEP INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	1.000000	0.000000	0.000000
.1	1.000000	0.007554	0.088178
.2	1.000000	0.010742	0.125390
.3	1.000000	0.015321	0.178841
.4	1.000000	0.021888	0.255503
.5	1.000000	0.030905	0.360758
.6	1.000000	0.042709	0.321136
.7	1.000000	0.057527	0.419239
.8	1.000000	0.075487	0.521343
.9	1.000000	0.096628	0.613881
1.0	1.000000	0.120914	0.685596
1.1	1.000000	0.148240	0.855300
1.2	1.000000	0.178448	0.913894
1.3	1.000000	0.2113297	0.953478
1.4	1.000000	0.246639	0.980364
1.5	1.000000	0.284099	0.999999
2.0	1.000000	0.492739	1.000000
2.5	1.000000	0.707535	0.999999
3.0	1.000000	0.893903	0.981598
3.5	1.000000	1.030150	0.982565
4.0	1.000000	1.109276	0.976934
4.5	1.000000	1.136551	0.975941
5.0	1.000000	1.125026	0.978992
5.5	1.000000	1.090705	0.984588
6.0	1.000000	1.048565	0.991031
6.5	1.000000	1.010015	0.996905
7.0	1.000000	0.981849	1.001314
7.5	1.000000	0.956447	1.003909
8.0	1.000000	0.962781	1.004805
8.5	1.000000	0.967767	1.004402
9.0	1.000000	0.977573	1.003221
9.5	1.000000	0.988647	1.001766
10.0	1.000000	0.9983456	1.000430

TABLE XXIII

RESPONSES OF THIRD ORDER SYSTEM
WITH RAMP INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
.1	0.100000	0.000036	0.001089
.2	0.200000	0.000076	0.002320
.3	0.300000	0.000300	0.009177
.4	0.400000	0.000903	0.027645
.5	0.500000	0.002134	0.065329
.6	0.600000	0.004281	0.128117
.7	0.700000	0.007657	0.228185
.8	0.800000	0.012595	0.361063
.9	0.900000	0.019434	0.521166
1.0	1.000000	0.028511	0.698575
1.1	1.100000	0.040157	0.882963
1.2	1.200000	0.054684	1.055613
1.3	1.300000	0.072390	1.213789
1.4	1.400000	0.093543	1.354809
1.5	1.500000	0.118388	1.479742
1.6	1.600000	0.147136	1.591284
1.7	1.700000	0.179968	1.697413
1.8	1.800000	0.217030	1.799544
1.9	1.900000	0.258435	1.899988
2.0	2.000000	0.304260	2.000000
2.5	2.500000	0.600080	2.499999
3.0	3.000000	1.000013	3.000000
3.5	3.500000	1.483384	3.499997
4.0	4.000000	2.022071	3.999842
4.5	4.500000	2.587622	4.499555
5.0	5.000000	3.145492	4.999163
5.5	5.500000	3.712802	5.498732
6.0	6.000000	4.248793	5.998383
6.5	6.500000	4.763574	6.498111
7.0	7.000000	5.260957	6.997954
7.5	7.500000	5.747086	7.497906
8.0	8.000000	6.228398	7.997917
8.5	8.500000	6.710209	8.497985
9.0	9.000000	7.195996	8.998071
9.5	9.500000	7.687303	9.498145
10.0	10.000000	8.184053	9.998202

TABLE XIV
RESPONSES OF THIRD ORDER SYSTEM
WITH $t^2/2$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
.1	0.005000	0.000009	0.000456
.2	0.020000	0.000011	0.000588
.3	0.045000	0.000028	0.001395
.4	0.080000	0.000085	0.004291
.5	0.125000	0.0002324	0.011789
.6	0.180000	0.000548	0.026315
.7	0.245000	0.001138	0.055818
.8	0.320000	0.002142	0.104136
.9	0.405000	0.003732	0.175489
1.0	0.500000	0.006117	0.272266
1.1	0.605000	0.009535	0.396058
1.2	0.720000	0.014261	0.539796
1.3	0.845000	0.020595	0.700306
1.4	0.980000	0.028871	0.872563
1.5	1.125000	0.039445	1.051431
1.6	1.280000	0.052697	1.232552
1.7	1.445000	0.069027	1.417555
1.8	1.620000	0.088850	1.605994
1.9	1.805000	0.112595	1.798184
2.0	2.000000	0.140702	1.997016
2.1	2.205000	0.173613	2.204176
2.2	2.420000	0.211776	2.419788
2.3	2.645000	0.255639	2.644448
2.4	2.880000	0.305643	2.879982
2.5	3.125000	0.362225	3.125000
3.0	4.500000	0.758278	4.500002
3.5	6.125000	1.376219	6.125005
4.0	8.000000	2.250892	8.000006
4.5	10.125000	3.402730	10.125012
5.0	12.500000	4.839004	12.500303
5.5	15.125000	6.557061	15.125464
6.0	18.000000	8.548361	18.001165
6.5	21.125000	10.802278	21.126839
7.0	24.500000	13.309015	24.502816
7.5	28.125000	16.061357	28.125457
8.0	32.000000	19.055308	32.004428
8.5	36.125000	22.289854	36.130360
9.0	40.500000	25.766196	40.506310
9.5	45.125000	29.486784	45.132150
10.0	50.000000	33.454415	50.007890

TABLE XXV

RESPONSES OF THIRD ORDER SYSTEM WITH $t^3/3!$ INPUT

Time (Sec.)	Command Signal	Uncompensated Response	Compensated Response
0.0	0.000000	0.000000	0.000000
.1	0.000167	0.000003	0.000148
.2	0.001333	0.000003	0.000138
.3	0.004500	0.000003	0.000166
.4	0.010667	0.000007	0.000369
.5	0.020833	0.000021	0.001076
.6	0.036000	0.000057	0.002521
.7	0.057167	0.000137	0.006741
.8	0.085333	0.000296	0.014936
.9	0.121500	0.000583	0.029305
1.0	0.166667	0.001067	0.052459
1.1	0.221833	0.001839	0.087603
1.2	0.288000	0.003016	0.136594
1.3	0.366167	0.004743	0.202364
1.4	0.457333	0.007198	0.286894
1.5	0.562500	0.010592	0.391627
1.6	0.682667	0.015717	0.517487
1.7	0.818833	0.021233	0.664831
1.8	0.972000	0.029096	0.833565
1.9	1.143167	0.039133	1.023331
2.0	1.333333	0.051759	1.233593
2.1	1.543500	0.067433	1.463506
2.2	1.774667	0.086657	1.713433
2.3	2.027833	0.109978	1.983081
2.4	2.304000	0.137989	2.272899
2.5	2.604167	0.171326	2.583763
2.6	2.929333	0.210668	2.917059
2.7	3.280500	0.256735	3.273690
2.8	3.658667	0.310292	3.655428
2.9	4.064833	0.372138	4.063725
3.0	4.500000	0.443113	4.500000
3.5	7.145833	0.966659	7.145833
4.0	10.666667	1.862206	10.666679
4.5	15.187500	3.263823	15.187522
5.0	20.833333	5.312400	20.831440
5.5	27.729167	8.149825	27.729200
6.0	36.000000	11.915013	36.018430
6.5	45.770833	16.741949	45.843507
7.0	57.166667	22.759412	57.349459
7.5	70.312500	30.091879	70.679660
8.0	85.333333	38.861019	85.973110
8.5	102.354170	49.187280	103.363805
9.0	121.500000	61.191170	122.977785
9.5	142.895830	74.994170	144.943200
10.0	166.666667	90.719130	169.380550

TABLE XXVI

(A) AMPLIFIER GAINS DESIGNED ON SLIDE-RULE ACCURACY

Amplifier	Polarity	Gains
B ₀	(+)	10.6
B ₁	(-)	20.1
B ₂	(+)	13.2
B ₃	(-)	2.7

(B) COMPENSATED RESPONSE

Time (Sec.)	Desired Response	Compensated Response
0.0	0.000000	0.000000
0.5	0.333333	0.327593
1.0	0.666667	0.660496
1.5	1.000000	0.989030
2.0	1.000000	1.025258
2.5	1.000000	1.019463
3.0	1.000000	0.991001
3.5	1.000000	0.961212
4.0	1.000000	0.941473
4.5	1.000000	0.935446
5.0	1.000000	0.941637
5.5	1.000000	0.955870
6.0	1.000000	0.973289
6.5	1.000000	0.989730
7.0	1.000000	1.002447
7.5	1.000000	1.010246
8.0	1.000000	1.013268
8.5	1.000000	1.012532
9.0	1.000000	1.009459
9.5	1.000000	1.005457
10.0	1.000000	1.001657

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